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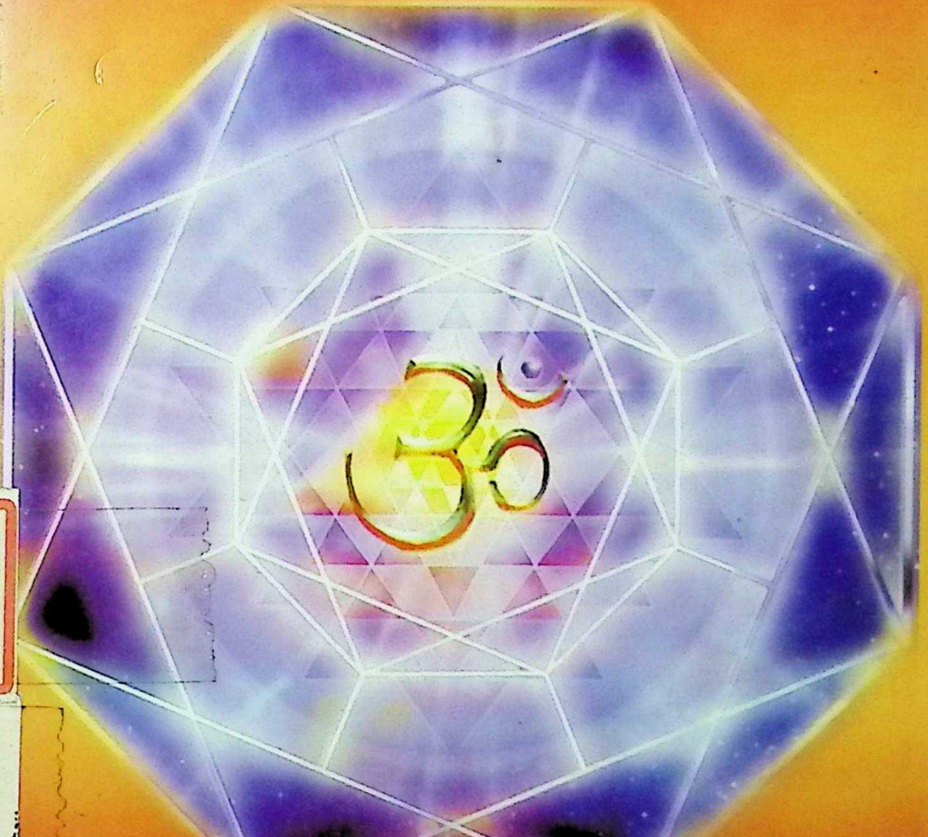
VEDIC MATHEMATICS



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श्वविद्यालय, हरिद्वार
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paedia of vedic
ematics [Vol-IV]
दिनांक | सदस्य संख्या
हरिद्वार

ISBN 978-81-261-3504-2 (Set)

पुस्तकालय
गुरुकुल काँगड़ी विश्वविद्यालय, हरिद्वार

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वर्ग संख्या 547-B	आगत संख्या 144581

पुस्तक विवरण की तिथि नीचे अंकित है। इस तिथि सहित ३० वें दिन यह पुस्तक पुस्तकालय में वापस आ जानी चाहिए अन्यथा ५० पैसे प्रतिदिन के हिसाब से विलम्ब दण्ड लगेगा।

Stock Verification-2024

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ENCYCLOPAEDIA OF VEDIC MATHEMATICS

(Vol. 4)

Shyam Priya

144581

144581

ANMOL PUBLICATIONS PVT. LTD.
NEW DELHI - 110 002 (INDIA)

ANMOL PUBLICATIONS PVT. LTD.
H.O.: 4374/4B, Ansari Road, Darya Ganj,
New Delhi-110 002 (India)
Ph.: 23278000, 23261597
B.O.: No. 1015, Ist Main Road, BSK IIIrd Stage
IIIrd Phase, IIIrd Block,
Bangalore - 560 085 (India)
Visit us at: www.anmolpublications.com

Encyclopaedia of Vedic Mathematics

© Author

First Published, 2008
ISBN 978-81-261-3504-2 (Set)

R510
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PRINTED IN INDIA

Printed at Mehra Offset Press, Delhi.

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Preface

Vedic Mathematics contains many examples of striking methods of calculation and there is a remarkable coherence to the system which makes it very attractive. Also the Vedic system itself suggests a kind of approach that involves going directly to the answer. Vedic mathematics is based on sixteen Sutras and some sub-Sutras which provide links through mathematics: the word 'Sutra' means 'thread'.

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—***Shyam Priya***

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Vedic Mathematicians in the Celestial Timekeepers

Introduction

The Ancient Vedics seemed to have an obsession for precision as well as a fascination for large numbers. A combination such as this makes an excellent prerequisite for time keeping and for devising a useful and practical calendar. So, they turned to the sky and began to decipher the meaning behind the various cycles they observed. Let us see how they went about developing a calendar that would convey a lot of information merely by knowing the day of the month after constant observation of the sky both during the day and the night over centuries.

The basic information they used for purposes of time keeping were the motions of the sun and the moon relative to the earth. So far nothing unusual, as did all the other ancients. The cycles they used apart from the day, the week, the fortnight, and the month are shown in Table.

Some Definitions

Let us establish the coordinate systems first. Everyday the celestial sphere appears to turn as the earth rotates, causing the daily rising and setting of the sun, stars and other celestial objects.

1. 60 year Jovian cycle/360 year 'divine cycle
2. 2700 year cycle of the Sapta Rishi or the Ursa Major
3. 27000 year cycle of the asterisms called the Great Year or the precession cycle
4. 432,000 year cycle called a yuga (= duration of Kaliyuga)
5. 4,320,000 year cycle known as the Maha Yuga
6. Kalpa, the cycle consisting of 4.32*10**9 years

Cosmology and Numerology

A bit of trivia - all of the numbers in Table 1 are divisible by 9 except the 60 year cycle. The number 9 and its multiples have a mystic significance in the Vedic tradition.

One way of visualizing the number 9 is as follows

The universe is constituted of 3 factors - time, space, and causation

The universe is constituted of 3 Gunas (ingredients) — Sattva, rajas, and tamas

The universe is constituted of the three functions — creation, preservation, and destruction

Thus 3 times 3 makes 9

Two times 9 makes 18, the number of chapters in the Bhagawad Gita

The 18 portions (parvas) of the Mahabharata epic define in detail the career of man on earth.

Thee are 18 days of warfare in the Great Bharata War

There are a total of 18 divisions in the Mahabharata war. 7 divisions on the Pandava side and 11 on the Kaurava side.

The Mahabharata war is thus an exposition of the human possibilities and achievements graded into eighteen categories, the first multiple of 9.

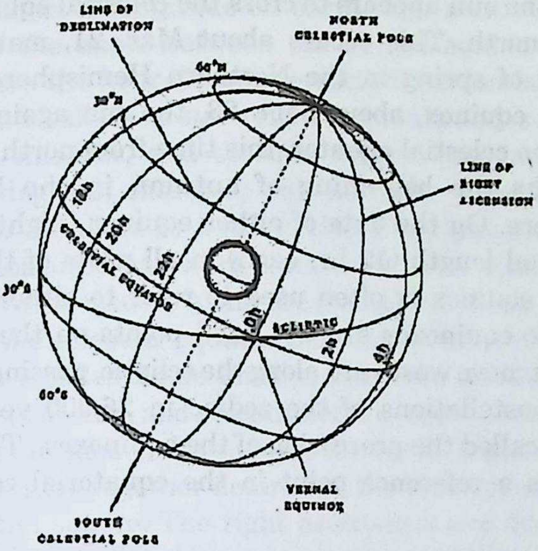
LINE OF DECLINATION
NORTH CELESTIAL POLE
60°N
30°N
LINE OF RIGHT ASCENSION
18h
20h
CELESTIAL EQUATOR
22h
30°S
0h
ECLIPTIC
2h
4h
60°S
VERNAL EQUINOX
SOUTH CELESTIAL POLE

Figure 1 The celestial sphere showing the ecliptic and its inclination to the celestial equator

Ecliptic

The great circle on the celestial sphere that lies in the plane of the earth's orbit (called the plane of the ecliptic). Because of the earth's yearly revolution around the sun, the sun appears to move in an annual journey through the heavens with the ecliptic as its path. The ecliptic is the principal axis in the ecliptic coordinate system . The two points at which the ecliptic crosses the celestial equator are the equinoxes. The obliquity of the ecliptic is the inclination of the plane of the ecliptic to the plane of the celestial equator, an angle of about 23 1/2°. The constellations through which the ecliptic passes are the constellations of the zodiac.

Equinox (Vasanth Sampat) Vernal Equinox

Either of two points on the celestial sphere where the ecliptic and the celestial equator intersect. The vernal equinox, also known as "the first point of Aries," is the point at which the sun appears to cross the celestial equator from south to north. This occurs about Mar. 21, marking the beginning of spring in the Northern Hemisphere. At the autumnal equinox, about Sept. 23, the sun again appears to cross the celestial equator, this time from north to south; this marks the beginning of autumn in the Northern Hemisphere. On the date of either equinox, night and day are of equal length (12 hr. each) in all parts of the world; the word equinox is often used to refer to either of these dates. The equinoxes are not fixed points on the celestial sphere but move westward along the ecliptic, passing through all the constellations of the zodiac in 26,000 years. This motion is called the precession of the equinoxes . The vernal equinox is a reference point in the equatorial coordinate system.

Equatorial Coordinate System

the most commonly used astronomical coordinate system for indicating the positions of stars or other celestial objects on the celestial sphere. The celestial sphere is an imaginary sphere with the observer at its centre. It represents the entire sky; all celestial objects other than the earth are imagined as being located on its inside surface. If the earth's axis is extended, the points where it intersects the celestial sphere are called the celestial poles; the north celestial pole is directly above the earth's North Pole, and the south celestial pole directly above the earth's South Pole.

The great circle on the celestial sphere halfway between the celestial poles is called the celestial equator; it can be thought of as the earth's equator projected onto the celestial sphere. It divides the celestial sphere into the northern and southern skies. An important reference point on the celestial equator is the vernal equinox , the point at which the sun crosses the celestial equator in March. To designate the position of a star, the astronomer considers an imaginary great circle passing through the celestial poles and through the star in question. This is the star's hour circle , analogous to a meridian of longitude on earth. The astronomer then measures the angle between the vernal equinox and the point where the hour circle intersects the celestial equator. This angle is called the star's right ascension and is measured in hours, minutes, and seconds rather than in the more familiar degrees, minutes, and seconds. (There are 360 degrees or 24 hours in a full circle.) The right ascension is always measured eastward from the vernal equinox. Next the observer measures along the star's hour circle the angle between the celestial equator and the position of the star. This angle is called the declination of the star and is measured in degrees, minutes, and seconds north or south of the celestial equator, analogous to latitude on the earth. Right ascension and declination together determine the location of a star on the celestial sphere. The right ascensions and declinations of many stars are listed in various reference tables published for astronomers and navigators. Because a star's position may change slightly (see proper motion and precession of the equinoxes), such tables must be revised at regular intervals. By definition, the vernal equinox is located at right ascension 0 h and declination 0°.

Another useful reference point is the sigma point, the point where the observer's celestial meridian intersects the celestial equator. The right ascension of the sigma point is equal to the observer's local sidereal time . The angular distance from the sigma point to a star's hour circle is called its hour angle ; it is equal to the star's right ascension minus the local sidereal time. Because the vernal equinox is not always visible in the night sky (especially in the spring), whereas the sigma point is always visible, the hour angle is used in actually locating a body in the sky.

Calendars and Tithis

Like most Asian calendars Indian calendars do not employ the solar year and day (i. e. tropical year and solar day) but the sidereal year, and the Synodic month(29.5306 days). Thus, the calendric year based on the sidereal year is defined as the time between two successive passes of the sun through a certain star's circle of declination. Lunar days and sidereal months are also used, and in certain lunisolar calendars lunar year and lunar month are taken into account, too.

Astronomical knowledge of Ancient India was written down in scientific treatises, called Siddhantas. In them, values for the lengths of months and years were given representing the latest knowledge at the time the Siddhanta was written. The values range from 365.258681 days in the Âryabhatiya to 365.258756 days in the Surya Siddhanta and are all too long compared with the modern sidereal year length of 365.25636 days. Nevertheless they are still in use for Indian calendars today.

The sidereal month is about two day shorter (27.3217) than the Synodic month.

Meaning of Tithi

According to the Indian calendar or Panchanga, Tithi is a lunar date based on the rotation of the moon around the earth, and is one of the five important aspects of an Indian almanac (Panchanga — Panch means five and anga means parts). Most of the Indian social and religious festivals are celebrated on a date corresponding to the original Ṭithi.

The current calendar "date" that we are so familiar with in our daily life is heliocentric and is based on the rotation of the earth around the sun. It takes the earth approximately 365 ¼ days to complete its rotation around the Sun. The calendar that most of us use today divides the 365 days of earth's period of rotation around the Sun in twelve months. The leap year, which occurs once every four years, accounts for ¼ day per year.

Similar to the solar calendar, the lunar calendar is also popular and widely used in the Asian countries such as

China, Pacific-rim countries, Middle East countries, and India. The lunar calendar, which is believed to have originated in India, has been around for a very long time, even long before the solar calendar.

The lunar calendar is geocentric and is based on the moon's rotation around the Earth. The lunar month corresponds to one complete rotation of the Moon around the Earth. Since this period of rotation of moon around the earth varies, the duration of lunar month also varies. On average, the lunar month has about 29 ½ days, the period of the lunar Synodic orbit. In addition to moon's rotation around the earth, the lunar year is based on earth's rotation around the Sun. In general, the lunar year has twelve lunar months of approximately 354 days (29.5 *12), thus making it shorter by about 11 days than the solar year. However, the lunar calendar accounts for this difference by adding an extra lunar month about once every 2 ½ years. The extra lunar month is commonly known as "Adhik Mas" in India (Adhik means extra and the Mas means month). The concept of this extra month is similar to the "Blue Moon" in the West, which occurs almost with the same frequency of 2 ½ years.

The Indian lunar year begins on the new moon day that occurs near the beginning of the Spring season. The twelve lunar months are:

Chaitra
Vaishakh
Jeshta
Ashadh
Shrawan(Sawan)
Bhadrapad(Bhado)
Ashwin
Kartik
Margshirsh

Paush
Magha
Falgoon (Fagan)

As mentioned earlier, to account for the difference between the solar and lunar year an extra lunar month occurs about every 2 ½ years as "Adhik Mas".

According to the Moslem calendar which is widely followed in Middle East and in other Moslem countries the lunar year is strictly based on twelve lunar months of 354 days per year. That's why their holy month of Ramadan occurs by approximately 11 to 12 days earlier than that in the preceding year.

The solar day (commonly referred as the "the date" in western calendar) has a fixed length of 24 hours. The change of date occurs at midnight as per local time or standard time of a given local time zone. Thus, the date changes from midnight to midnight. Similarly the day (as in weekdays) changes from midnight to midnight as per local or standard time for that location. In other words, as per the western (or English) calendar the length of day and date is exactly 24 hours, and there is a definite correspondence between the date and the corresponding day of the week.

A lunar day usually begins at sunrise, and the length of lunar day is determined by the time elapsed between the successive sunrises. As per the Jewish calendar their lunar day begins at the sunset, and lasts through the next sunset. A lunar day is essentially the same as a weekday. In India the lunar day is commonly referred as "War". Just as the English calendar has seven days for a week, the Indian calendar has seven wars for a week. Thus,

The lunar day, however, varies approximately between 22 to 26 hours based on the angular rotation of moon around the earth in its elliptical orbit. In the Indian calendar, the lunar date is referred as "Tithi". The basis for the length of a lunar date is geocentric and is defined as the angular distance between the sun and the moon as seen from the earth. As the moon rotates around the earth, the relative angular distance between the sun and the moon as seen from the earth increases from 0 degrees to 360 degrees. It takes one lunar month or about 29 ½ solar days for the angular distance between the sun and the moon to change from 0 to 360 degrees. When the angular distance reaches zero, the next lunar month begins. Thus, at the new moon a lunar month begins, at full moon, the angular distance between the sun and the moon as seen from the earth becomes exactly 180 degrees.

English calendar weekdays	***Indian calendar weekdays***
Sunday	
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	Raviwar
Somwar (Chandrawar)	
Mangalwar	
Budhwar	
Guruwar	
Shukrawar	
Shaniwar	

The lunar cycle begins with crescent moon and the crescent phase lasts till that phase culminates in the full moon, typically lasting for about 15 days. Then the moon enters in the waning phase until it disappears from the sky by lining up with the Sun. The waning phase also lasts for about 15 days. According Indian lunar month, the crescent lunar phase fortnight is called as "Shudha or Shukla Paksha" and the waning phase of the lunar cycle fortnight as "

Krishna Paksha". Thus, during Shudha (or Shukla) Paksha the angular distance between the moon and the sun varies from 0 degrees to 180 degrees while that during the Krishna Paksha from 180 to 0 degrees. If we divide 180 degrees into 15 equal parts, then each part becomes of 12 degrees in length. Thus, this each twelve-degree portion of angular distance between the moon and the sun as it appears from the earth is the lunar date or Tithi. Tithis or lunar dates in Shudha (or Shukla) Paksha begin with Prathama (first), Dwitiya (second), etc., till we reach the Poornima, the lunar date for full moon day. Similarly for the waning fortnight lunar cycle or Wadya (or Krushna) Paksha, this begin again with Prathama (first), Dwitiya (second), etc., till we arrive Amavasya or a day before the new moon. Thus when we refer to Ramnavami (the birthday of Rama), it's the Navami (ninth lunar day) of Shudha Paksha of the lunar month Chaitra, or Chaitra Shudha Navami. Similarly, the Gokulashtmi (also called as Janmashtami, the birthday of Krishna) occurs on Shrawan Wadya Ashtami (eighth lunar day of Wadya Paksha of the lunar month Shrawan).

The angular velocity of moon in its elliptical orbit around the earth varies continuously as it is affected (according to Kepler's Law) by the relative distance between the earth and the moon, and also by the earth's relative distance from the sun. As a result, the daily angular speed (the speed of the angular change between the moon and the sun as seen from the earth) varies somewhere between 10 to 14 degrees per day. Since the length of a Tithi corresponds to 12 such degrees, the length of a Tithi also varies accordingly. Therefore, a Tithi can extend over one day (24 hour·period) or it can get sorteneded if two Tithis occur in one 24 hour day.

Since the angular distance between the moon and the sun as referred here is always relative to the entire earth, a lunar day or Tithi starts the same time everywhere in the world but not necessarily on the same day. Thus, when a certain Tithi starts at 10:30 PM in India it also begins in

New York at the same time, which is 12 PM (EST) on the same day. Since the length of a Tithi can vary between 20 to 28 hours, its correspondence to a War (a weekday) becomes little confusing.

As per the Indian calendar, the Tithi for a given location on the earth depends on the angular distance between the moon and the sun relative to the earth at the time of sunrise at that location. Thus, for instance, assume on a November Monday sunrise in New York city occurs 8:30 AM (EST). Further assume that at 9 AM (EST) on Monday the angular distance between the sun and moon is exactly 12 degrees just following the new moon of the Indian lunar month Kartik. Since the length of a tithi is 12 degrees, the tithi, Kartik Shudha Dwitiya (second day) begins exactly at 9 AM on Monday of that November in New York. However, at the time of sunrise on that Monday the tithi Dwitiya has not begun. Therefore, the tithi for that Monday for city of New York is Kartik Shudha Prathama (first day). On the same Monday morning the sunrise in Los Angeles occurs well past 9 AM (EST). Since the Tithi Dwitiya occurs everywhere in the world at the same instant, therefore, for Los Angeles, the Tithi for that Monday would be Karthik Shudha Dwitiya.

For the same Monday at 9 AM (EST), it would be 7:30 PM in Mumbai or New Delhi. Thus, Tithi for that Monday for city of New York, Mumbai, and New Delhi is Karthik Shudha Prathama (the first day of Indian lunar month Karthik) while for most of the regions west of Chicago or St. Louis the Tithi for that Monday is Dwitiya. In other words, the Tithi Karthik Shudha Prathama for regions west of Chicago or St. Louis should occur on the preceding day, the Sunday.

Karthik Shudha Prathama (the first day of Indian lunar month Karthik) also happens to be the first day after Diwali. Most of the Indians celebrate this as their New Year 's Day. Indians living in India, Europe, and eastern part of the

United States thus should celebrate their New Year on that Monday while regions west of Chicago should celebrate on the preceding day, the Sunday. (Based on description by Jagdish C. Maheshri, October 12, 2000).

Adhik Mas Occurs only when Two Amavasyas

1	Pratipat	Pratipat	The presiding deity of the first lunar day in Brahma and is good for all types of auspicious and religious ceremonies
2	Dvitiya	Dvitiya	Vidhatr rules this lunar day and is good for the laying of foundations for buildings and other things of a permanent nature.
3	Trtiya	Trtiya	Visnu is the lord of this day and is good for the cuttings of one's hair and nails and shaving.
4	Caturthi	Caturthi	Yama is lord of the 4th lunar day, which is good for the destruction of one's enemies, the removal of obstacles, and acts of combat.
5	Pancami	Pancami	The Moon rules this day, which is favourable for administering medicine, the purging of poisons, and surgery.
6	Sasti	Sasti	Karttikeya presides over this day and is favourable for coronations, meeting new friends, festivities, and enjoyment.
7	Saptami	Saptami	The 7th lunar day is ruled by Indra; one may begin a journey, buy conveyances, and deal with other such things as a movable nature.
8	Astami	Astami	The Vasus rule this day, which is good for taking up arms, building of one's defenses, and fortification.
9	Navami	Navami	The Serpent rules this day, with is suitable for killing enemies, acts of destruction, and violence.

10	Dasami	Dasami	The day is ruled by Dharma and is auspicious for acts of virtue, religious functions, spiritual practices, and other pious activities.
11	Ekadasi	Ekadasi	Rudra rules this day; fasting, devotional activities, and remembrance of the Supreme Lord are very favourable.
12	Dvadasi	Dvadasi	The Sun rules this day, which is auspicious for religious ceremonies the lighting of the sacred fire, and the performance of one's duties.
13	Trayodasi	Trayodasi	The day is ruled by Cupid and is good for forming friendships, sensual pleasures, and festivities.
14	Caturdasi	Caturdasi	Kali rules this day suitable for administering poison and calling of elementals and spirits.
15	Amavasya (new moon)	Purnima (full moon)	The Vasve-devas rule the New Moon suitable for the propitiation of the Manes and performance of austerities.

Zodiac sign	*Sanskrit Name*	*a*	*Sector end*
Aries	Mesha	00	30
Taurus	Vrishabha	30	60
Gemini	Mithuna	60	90
Cancer	Karka	90	120
Leo	Simha	120	150
Virgo	Kanya	150	180
Libra	Tula	180	210
Scorpio	Vrishchika	210	240
Sagittarius	Dhanus	240	270
Capricorn	Makara	270	300
Aquarius	Kumbha	300	330
Pisces	Meena	330	360

The Tropical Zodiac

Ecliptic, Tropical Zodiac and the Sidereal Zodiac

9 degrees to either side of the Ecliptic is a belt of the Heavens known as the Zodiac. (Dante called it the Oblique Line that beareth all planets).

First 30 degrees of the Zodiac constitute the sign of Aries. The next 30 degrees Taurus and so on. The Zodiac counted from the first degree of Aries to the 360th degree of Pisces is called the Tropical Zodiac.

These 12 signs are the limbs of the Cosmic Man or Time Eternal (Kalapurusha- The Almighty Self as Time).

Aries is His head, Taurus His face, Gemini His neck, Cancer His heart, Leo the place beneath, Virgo His belly, Libra His generative organs, Scorpio the place beneath, Sagittarius His upper thigh, Capricorn his lower thigh, Aquarius His leg and Pisces His feet!

The Clock, the Sidereal Zodiac, Nakshatras, and the Precession of the Equinoxes

The basis of the Hindu calendar calculation is Vedic. This calendar has been modified and elaborated, but because it is based on the stars (Nakshatras) visible to the naked eye, and on the visible Lunar phases, it is more accurate than any others of the past. The actual moments when Lunar months begin can easily be checked by the regular appearances of Solar eclipses, and the middle moment of a Lunar month — Poornima or full moon -- can similarly be verified by the more frequent Lunar eclipses. Hence the Hindu calendar, not requiring special instruments for its rectification, has maintained great accuracy for thousands of years.

The oldest calendar is probably the Vedic among the languages referred to as i.e languages; at first lunar, later with solar elements added to it. The sister Avesta calendar is similarly first Lunar, but later only Solar. Both these calendars (the oldest in the i.e universe) are influenced by the prehistoric calendars of the first and second root races at the North Pole and its surroundings, as they reckon with days and nights lasting six months.

For untold ages, the Hindus have observed the motion of the moon, the sun and the seven planets along a definite path that circles our sky and is marked by fixed clusters of stars. The moon afforded the simplest example. These early astronomers observed that the moon, moving among these fixed star constellations which they called Nakshatras, returned to the same Nakshatra in 27.32166 days, the exact quantity determined by Aryabhatta, thus completing one Nakshatra month. They found it convenient to divide these groups of stars into 27 almost equal sections, or the 27 Nakshatras. By this method of reckoning, instead of giving the date of a month, as Western calendars do, the Hindus gave the name of the Nakshatra in which the moon was to be seen. (The moon is in each of these Nakshatras for approximately one day plus eighteen minutes.)

This scheme fitted nicely with the sun's cycle, for the Hindus noted that the sun traversed the same circle through the sky, but that it returned to its starting place only after 365.258756481 days, or what we call a Solar Sidereal Year. (Modern figures based on this Hindu figure quote 365.2596296 days -- a distinction without a difference, for ordinary purposes.) Now, having already divided the month into the 27 Nakshatras for the convenience of reckoning the moon's voyage through the heavens, what more natural than that these same Nakshatras should serve for the study of the Sun's course? Being in a circle of 360 degrees, each Nakshatra takes up 13 1/3 degrees of that circle. The Sun, moving about 1 degree in a day, is seen for 13 1/3 days in each Nakshatra. The system of reckoning according to the moon Nakshatras is current today that of the sun's being uncommon.

During the course of one day, the earth has moved a short distance along its orbit around the sun, and so must rotate a small extra angular distance before the sun reaches its highest point. The stars, however, are so far away that the earth's movement along its orbit makes a generally negligible difference to their apparent direction (see, however parallax), and so they return to their highest point in slightly less than 24 hours. A mean sidereal day is about 23h 56m in length. Due to variations in the rotation rate of the Earth, however, the rate of an ideal sidereal clock deviates from any simple multiple of a civil clock. The actual period of the Moon's orbit as measured in a fixed frame of reference is known as a Sidereal month, because it is the time it takes the Moon to return to the same position on the celestial sphere among the fixed stars (Latin: sidus): 27.321 661 days (27 d 7 h 43 min 11.5 s) or about 27 days. This type of month has appeared among cultures in the Middle East, India, and China in the following way: they divided the sky in 27 or 28 lunar mansions or Nakshatras, characterized by asterisms (apparent groups of stars), one for each day that the Moon follows its track among the stars.

The basis of the Hindu calendar calculation is Vedic. This calendar has been modified and elaborated, but because it is based on the stars (Nakshatras) visible to the naked eye, and on the visible Lunar phases, it is more accurate than any others of the past. The actual moments when Lunar months begin can easily be checked by the regular appearances of Solar eclipses, and the middle moment of a Lunar month — Purnima or full moon -- can similarly be verified by the more frequent Lunar eclipses. Hence the Hindu calendar, not requiring special instruments for its rectification, has maintained great accuracy for thousands of years.

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In brief, then, the earliest method, the Vedic, of counting, was to name the moon through the various Nakshatras — the circle or cycle repeating itself each Sidereal-Star-Month. Later the sun's place in the same Nakshatras was noted, the year ending when the Sun returned to the same Nakshatra. Then came the noting of the Solar and Lunar eclipses, and the observance of the New and Full Moons divided the month into the two phases of waxing and waning Moon, the month beginning at the moment of New Moon. This is how the Hindus reckon today, the month taking its name from the Nakshatra in which the Full Moon is seen each month. The Full Moon being exactly opposite the Sun, the Solar nakshatra bears the same name as the Lunar month six months ahead, while each Lunar month bears the same name as the 14th Solar Nakshatra ahead.

The Western student faced with these unfamiliar calculations may echo the old Persian proverb, "Why count big numbers and small fractions, when they are all amassed in 1?" But the Hindu looks on these figures from another point of view—he lives with them, and among them, and by them, much of the time. Consider a Sanskrit sloka (verse) about the Savati or pearl nakshatra, which marks the new season after the monsoon is over. The sloka says, "If in the Swati a rain drop falls into the sea, that drop becomes a pearl." This may sound foolish, for the peasant, though he live in the depth of the interior of India, knows that pearls come from the sea—even if he does not necessarily understand that these pearls grow inside the oyster. He does know, however, that if it rains at this period of the year, his crops will yield great wealth. And the pearl is synonymous with wealth among people who, if they have any money, invest it in jewellery, especially gold and pearls, rather than in the banks. (Poetically, rice, their staple food)

To summarize, the earth revolves around the Sun once in 365 days 5 hours 48 minutes and 46 seconds. Considered from the earth, the Sun appears to complete one round of the ecliptic during this period. This is the Tropical year. In the span of a tropical year, the earth regains its original angular position with the Sun. It is also called the Year of seasons since the occurrence, and timing, of seasons depends on the rotation of the earth around the sun. If, for example, we consider the revolution of the Sun around the earth from one vernal equinox (around 21st March, when the day and night all over the globe are equal) to the next vernal equinox, it takes one tropical year to do so.

However, if at the end of a tropical year from one vernal equinox to the next, we consider the position of the earth with reference to a fixed star of the zodiac, the earth appears to lie some 50.26 seconds of celestial longitude to the west of its original position. In order for the earth to attain the same position with respect to a fixed star after one revolution, it takes a time span of 365 days 6 hours 9 minutes and some 9.5 seconds. This duration of time is called a sidereal year. The sidereal year is just over 20 minutes longer than the tropical year; this time difference is equivalent to 50.26 seconds of celestial longitude.

Each year, the Vernal equinox will fall short by 50.26 seconds along the zodiac reckoned along the fixed stars. This continuous receding of the Vernal equinox along the zodiac is termed the Precession of the Equinoxes and it takes about 25776 years to make one complete revolution of the precessional motion of the earth's axis. Hipparchus regarded as the discoverer of the precession of the equinoxes in the west gave us either 28,000 or 28,173 years for one revolution.. Another figure given is 25,920 years for the precession cycle, These figures indicate that the mean value of 27,000 years given in the Vedic scriptures is reasonable. The precession of the equinoxes has proved to be very useful for dating certain events in Vedic and Post Vedic times.

There are only a few methods, by which we can determine the age of an event in the absence of radiocarbon dating which is not as precise as the astronomical clocks.

1. Use the Precession of the equinoxes to determine the Nakshatra in which the Vernal equinox occurs in a particular Nakshatra. If, we recall there are 27 Nakshatras, it follows that the vernal equinox occurs in a different Nakshatra, once every 1000 years.
2. Use the statements made in the texts to check for internal consistency. If for example Aryabhatta uses a place value system, the zero must have been in fairly wide use by then. If further he uses classical sanskrit (codified by Panini then he must have lived after Panini.

Western Zodiac name	*Indian Nakshatras (Sidereal Zodiac)*	*Sector in deg,min deg, min*
1. Beta Arietis	Aswini	00 00 13 20
2. 41 Arietis	Bharani	13 20 26 40
3. Eta Tauri	Karthika	26 40 40 00
4. Alpha Tauri	Rohini	40 00 53 20
5. Lamda Orionis	Mrigasira	53 20 66 40
6. Alpha Orionis	Aridra	66 40 80 00
7. Beta Geminorum	Punarvasu	80 00 93 20
8. Delta Cancri	Pushya	93 20 106 40
9. Alpha Cancri	Aslesha	106 40 120 00
10. Alpha Leonis	Magha	120 00 133 20
11. Delta Leonis	Pubba	133 20 146 40
12. Beta Leonis	Uttara	146 40 160 00
13. Delta Corvi	Hasta	160 00 173 20
14. Alpha Virginis	Chitra	173 20 186 40
15. Alpha Bootis	Chothi	186 40 200 00
16. Beta Librae	Vishakam	200 00 213 20
17. Delta Scorpi	Anuradha	213 20 226 40
18. Alpha Scorpi	Jyeshta	226 40 240 00
19. Lamda Scorpi	Moola	240 00 253 20
20. Delta Sagittari	Poorvashad	253 20 266 40
21. Delta Sagittari	Uthrashad	266 40 280 00
22. Alpha Aquilae	Sravana	280 00 293 20

R
570
Shy-G

Contd...

Western Zodiac name	*Indian Nakshatras (Sidereal Zodiac)*	*Sector in deg,min deg, .nin*
23. Alpha Delphini	Dhanishta	293 20 306 40
24. Lamda Aquar	Satabhisha	306 40 320 00
25. Alpha Pegasi	Poorvabhadra	320 00 333 20
26. Alpha Andromeda	Uttrarabhadra	333 20 346 40
27. Zeta Piscium	Revathi	346 40 360 00

Values for the Lunar sidereal orbit and the Lunar Synodic orbit are given in Table below

Comparisons	*Lunar sidereal orbit*	*Lunar synodic orbit*
AD 2000.0	27.32166156	29.53058888
AD 498	27.3216638	29.530591
Aryabhatta	27.321668	29.530582
Paulisa Siddhanta	27.321673	29.530587
1604 BC	27.321668	29.530595
ASTRONOMIC		
AUTHORITY	Aryabhatta	
(from Clarke and Kay)		Surya
Siddanta		
Years in Cycle	4,320,000	4,320,000
Rotations	1,582,237,500	1,582,237,828
Days	1,577,917,500	1,577,917,828
Lunar Orbits	57,753,336	57,753,336
Kay notes 57,753,339 lunar orbits rather than 57,753,336 per Clarke.		
Synodic Months	53,433,336	53,433,336
Mercury	17,937,920	17,937,060
Venus	7,022,388	7,022,376
Mars	2,296,824	2,296,832
Jupiter	364,224	364,220
Saturn	146,564	146,568

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How old is the Universe, Kalachakra and the Yuga Concept, Hindu Cosmological Time Frames

The Hindu Calendar (also known as the Panchanga) currently in practice reckons time in terms of very large cycles called Kalpa (4.32 billion years) consisting of 14 Manvantaras (Manvantara or age of Manu,~ 308 million years). A Manvantara is made up of Mahayugas (Mahayuga= great yuga consists of 4 yugas: Krita, Treta, Dwapara and Kali). Kali yuga is equivalent to 432,000 years and 1 Mahayuga= 4.32 million years. This system appears to have been in use since the days of the Epics and Puranas, and attested in the Siddhantas. However, the earliest Vedic Calendar was based on a cycle also called yuga, but consisting of only five years. This ancient Vedic Calendar was a Luni-solar calendar and used two intercalary months in a five year period and has often been criticized as being very crude.

First we have Kalpa, a day in Brahma's 'life' or 4320 million earthly years, and a night of equal length. During the day he creates and during the night he absorbs to begin the cycle each Brahma day . Each kalpa is divided into 14 Manvantaras or 308.448 million years we are supposed to be in the seventh Manvantara of Vaivasvata Manu. Each Manvantara contains 71 Mahayugas , plus 1Krtayuga ,and each Mahayuga is divided into 4 yugas - Krta, Treta, Dvapara and Kali of 4800, 3600, 2400 and 1200 divine years of the Gods, each of which = 360 human years. We are at present in the Kali yuga which began in 3102 BCE the traditional year of the Mahabharata war .

Thus, we have a day in Brahma's life of 1 Kalpa

1 Brahma Day (day and night) = 2 Kalpa

1 Kalpa = 4,320,000,000 earthly years (Y)

= 14 Manus + 1Kritayuga = 1000 MY =14*71.4+.4

Kaliyuga = 432,000 Y = 1KY

= 1200 divine years (DY) = 1 Yuga

1 DY = 360 Y

Dwapara = 864,000 Y = 2KY = 2400 DY

TretaYuga = 1,296,000 Y = 3KY = 3600 DY

Kritayuga = 1,728,000 Y =4 KY = 4800 DY

= 0.4 MY =.4/71.4 = 5.6022408964e-3

Mahayuga (MY) = 4,320,000 earthly years

= 10 KY = 12000 DY

1Manvantra (M) = 71 MY = 306.72 million years

1 Manu = 1M + 1 KritaYuga = 308.448 million years

= 856,800 DY

1 Kalpa = 14 Manus + 1Krita Yuga = 14*71.4 +.4

= 1000 MY = 12,000,000 DY = 4.32 billion Y

So How Old is the Universe

As of Vaisakhapratipada of 2006 CE, May 1 we are in the second quarter of Brahma's day, called Shewtavarah Kalpa, seventh Manvantara named Vaivasvata and entered into the first quarter of the 28th Kaliyuga. Already 5107 years of this 28th KY have passed, so the time elapsed in this Kalpa is

6 M	=1,850,688,000 Y (6*308,448,000)
	= 6 M
And 27 MY	= 116,640,000 Y (27 * 4,320,000)
	= 27/71.4 = 0.3781512605 M
And 28th	
(Krita+Treta +Dwapara	= 3,888,000 Y (9*432,000)
	=0.9 MY =.9/71.4 = 0.012605042
M	
5107·Y of Kaliyuga)	= 5107 Y
So the current year 2006 CE	= 1 ,971,221,108 Y

The 12 signs of the Zodiac with Sanskrit names are mentioned In Brihat Samhita and Laghu Bhaskariyam. The former is the work of Varahamihira 505 CE. He is supposed to have borrowed it from a Greek of the 4th century CE. (Could it be Hipparchus)

To conclude this brief acquaintance with Vedic astronomy, we want to draw attention to the possible presence in the Rg-Veda of a momentous cultural artifact, the origin of which is usually situated in Babylonia in about 600 BC: the twelve-sign Zodiac. In RV 1:164:11, the sun wheel in heaven is said to have 12 spokes, and to be subdivided into 360 pairs of "sons": the days (consisting of day and night), rounded off to an arithmetically manageable number, also the basis of the "Babylonian" division of the circle in 3600. The division in 12 already suggests the Zodiac, and we also find, in the footsteps of N.R. Waradpande, that a number of the Zodiacal constellations/rashis (classically conceived as combinations of 2 or 3 successive Lunar mansions or Nakshatras of 13 ° and 20' each) are mentioned. Obviously the Rg should be dated prior to the beginning of Kaliyuga, as we have already demonstrated and hence the Babylonian origin of the twelve sign Zodiac is suspect.

India as a Creative Civilization

The idea of India as an imitative civilization that borrowed everything is based on colonial stereotyping and obsolete theories.

Unfounded Assumptions

It is a widespread belief, especially in the West, that Indian civilization developed mainly by borrowing from others and not through innovation and creative endeavour. This, however, is a recent view, one that can be traced to the motivated scholarship of the European colonial period and its immediate aftermath. The reality is that throughout history, except during the colonial period, peoples of all nations have looked to India as the most creative and original of civilizations. To be specific, medieval and ancient scholars from Arabia, Spain, China and even Greece- all acknowledged their indebtedness to Indian science. For example, a medieval Arab scholar Sa'id ibn Ahmad al-Andalusi (1029-1070) wrote in his Tabaqat al-'umam, one of the earliest books on history of sciences:

> *"The first nation to have cultivated science is India. ... India is known for the wisdom of its people. Over many centuries, all the kings of the past have recognized the ability of the Indians in all the branches of knowledge.*

"The kings of China have stated that the kings of the world are five in number and all the people of the world are their subjects. They mentioned the king of China, the king of India, the king of the Turks, the king of the Persians, and the king of the Romans.

> *"... They referred to the king of India as the "king of wisdom" because of the Indians' careful treatment of 'ulum [sciences] and all the branches of knowledge.*

"The Indians, known to all nations for many centuries, are the metal [essence] of wisdom, the source of fairness and objectivity. They are people of sublime pensiveness, universal apologues, and useful and rare inventions.

> *"... To their credit the Indians have made great strides in the study of numbers and of geometry. They have acquired immense information and reached the zenith in their knowledge of the movements of the stars [astronomy] ... After all that they have surpassed all other peoples in their knowledge of medical sciences ..."*

When the necessary allowance is made for the exuberance of the writer and even some exaggeration, it is clear that no one until the coming of the modern Europeans (and their Indian disciples) questioned the antiquity of Indian science.

In his book al-Andalusi goes on to give details of several Indian texts on astronomy and tells us that the Arab scholars used them in preparing their own almanacs.

Not only Medieval Arabs, even some early Christian scholars recognized Indian contributions. Writing in 662 AD, when the Byzantine Empire was its height and it was thought that there was no knowledge beyond Greek knowledge, Sebokht, the Bishop of Qinnesrin in North Syria observed:

> *"I will omit all discussion of the science of the Hindus [Indians], a people not the same as Syrians, their subtle discoveries in the science of astronomy, discoveries more ingenious than those of the Greeks and the Babylonians; their valuable method of calculation [the decimal system]; their computing that surpasses description. I wish only to say that this computation is done by means of nine signs. If those who believe because they speak Greek, that they have reached the limits of science should know these things, they would be convinced that there are also others who know something."*

The reference of course is to the famous place decimal system using zero invented by the Hindus. (It is often called the Arabic numeral system, but the Arabs themselves called it the Indian system acknowledging their indebtedness to India.) In fact the Greek (and the Roman) method of computing and solving equations was cumbersome in the extreme when compared to the method used by Indians. Mathematics as we know today would hardly be possible without this invention- probably the greatest single advance in the history of mathematics.

India's Contributions : Material

This brings up a basic issue about India's contribution: while the world has by and large acknowledged it in the spiritual realm - like Yoga, Vedanta and Buddhism - there is widespread belief that the Indians were other worldly and did not pay attention to material progress. This is far removed from the truth. In addition to mathematics and astronomy (more of which later), India led the world in ferrous and non-ferrous metallurgy, until the industrial base was deliberately destroyed by the British. Indian steel was deemed the best in the world well into the eighteenth century. (Steel is an Indian invention.) Indian textiles have been prized throughout history, until it was suppressed by the British, though it is now being gradually regained.

This being the case, why does this image of India as largely an imitative civilization still persist? (This though is mainly in the West- not Asia.) Three reasons can be given: (1) colonial stereotyping; (2) Marxist domination of intellectual life in the post-colonial period; and (3) incompetence of scholars.

The colonial image is gradually eroding but the Marxist formulation of India as an imitative culture is the result of what Marx himself wrote about India: "India has no history. What is called history is only the record of successive intruders." Marx of course knew nothing about India, but his statement became sacrosanct to Marxist scholars who continue to dominate the intellectual scene in India as elsewhere.

This brings us to the third cause, scholarly incompetence. Scholars of the colonial period whose works still constitute standard references on India, were mostly bureaucrats and missionaries and lacked the scientific knowledge needed to appreciate true achievements. This is generally true of history of science. Einstein himself complained more than once that most historians of science were linguists who really did not understand the problems scientists were grappling with. It is hardly surprising that charges of lack of scientific capacity and originality on the part of Indians is made by those who are themselves scientifically ignorant- like nineteenth century

European theologians and linguists continuing down to their modern Indian followers. It is interesting to contrast their views with those of competent scientists who studied ancient India- that is to say the views of people who were competent to express opinions about science. This will help shed light on the true fact of prejudice and unfounded charges used as a fig leaf to conceal their own scientific ignorance.

Albrecht Weber was a leading nineteenth century Sanskrit scholar from Germany. At a time when the relationship between Indian and Greek mathematics was being seriously debated, he went on to assert that the Vedic mathematics was borrowed by the Indians from the Greeks following Alexander's invasion. His actual statement was that there was "nothing of a literary-historical nature standing in the way of the assumption of a use [by the Vedic mathematicians] of the teachings of Hero of Alexandria." Hero of Alexandria is now known to have been living in 62 AD! Even in Weber's time Hero was known to be later than 200 BC. Now of course we know that Vedic mathematics must have existed long before 2000 BC. This is clear from archaeology.

There is more to this that goes to highlight the creativity and originality of Indian mathematics. After more than twenty years of research, the distinguished American mathematician and historian of science A. Seidenberg showed that Greek, Egyptian and Old-Babylonian mathematics are derivatives of the Vedic mathematics known as the Sulbasutras. In 1962, he reached the following epoch making conclusion:

> *"... the elements of ancient geometry found in Egypt [c. 2050-1800 BC] and Babylonia [c. 1900-1750 BC] stem from a ritual system of the kind observed in the Sulbasutras."*

Fifteen years later, in a famous paper, appropriately titled 'The Origin of Mathematics' Seidenberg elaborated further on his revolutionary discoveries: "The arithmetical tendencies here encountered [in Vedic mathematics] were expanded, and in connection with observations on the rectangle led to Babylonian mathematics.

> "*A contrary tendency, namely, a concern for exactness of thought (or the myth of its importance), together with a recognition that arithmetic methods are not exact, led to Pythagorian [Greek] mathematics.*"

In other words, both of the world's great systems of mathematics, which are seen as the basis of Western civilization, namely, Greek and Egyptian-Babylonian, owe their existence to ancient Indian (Vedic) mathematics. In the face of this to argue that Indian civilization did not originate anything is a colossal misrepresentation based on obsolete and discredited theories. In fact, it is well known that the Chinese themselves heavily borrowed from India in fields like astronomy and mathematics.

It is a similar story when we turn to astronomy. William Dwight Whitney and other linguists of the colonial period claimed that Indian astronomy was based on the Greek- a position still reiterated by many scholars. A.B. Keith, another linguist of the colonial period gave out the view that Indians had borrowed their astronomy from some 'Semitic source' without specifying which one. It is illuminating to contrast Keith's (and Whitney's) statements with what Jean-Sylvain Baily, a leading French astronomer had to say about the astronomical achievements of the Hindus:

> "... *the motions of the stars calculated by the Hindus before some 4500 years vary not even a single minute from the tables of Cassine and Meyer [used in Europe in the nineteenth century]. The Indian tables give the same annual variation of the moon as that discovered by Tycho Brahe—a variation unknown to the school of Alexandria and also to the Arabs who followed the calculations of the school. ... The Hindu systems of*

> *astronomy are by far the oldest and that, from which the Egyptians, Greeks, Romans and even the Jews derive the knowledge.*"

This, it is worth noting, is by a great astronomer- not some colonial bureaucrat or missionary scholar who knew some Indian language. And this view was echoed by John Playfair, Astronomer Royal at the University of Edinburgh.

India's Contributions II: Spiritual

In the spiritual realm, no one seriously disputes India's contributions, though even these are often distorted. While it is unnecessary to go into details of yoga, meditation and others, it is worth noting that even these have potential for concrete applications. For example, the great linguist Panini gave the concept for meta-language and constructed one thousands of years before computer scientists began exploring the same idea. No one has been able to match him to this day. Similarly, Patanjali, in his Yogasutra, gives rules for testing the correctness of proofs of statements and proof theories centuries before mathematicians began to ponder about these fundamental problems. It is worth noting that one of the central problems of modern physics (Quantum Physics) is the problem of reality and consciousness. This was anticipated by Vedic seers thousands of years ago. The great physicist J. Robert Oppenheimer wrote:

> "*The general notions about human understanding... which are illustrated by discoveries in atomic physics are not in the nature of things wholly unfamiliar, wholly unheard of or new. Even in our own culture they have a history, and in Buddhist and Hindu thought a more considerable and central place. What we shall find [in modern physics] is an exemplification, an encouragement, and a refinement of old wisdom.*"

In this context it is worth emphasizing that India's contribution of Buddhism to China (and other countries of the region) is by no means insubstantial. These civilizations would hardly exist without the Indian contribution in all aspects of culture- from science and technology, the arts, philosophy and spirituality. In his book The Art of Southeast Asia, Philip Rawson writes:

> *"The culture of India has been one of the world's most powerful civilizing forces. Countries of the Far East, including China, Korea, Japan, Tibet and Mongolia owe much of what is best in their own cultures to the inspiration of ideas imported from India. The West, too, has its own debts. But the members of that circle of civilizations beyond Burma scattered around the Gulf of Siam and the Java Sea, virtually owe their very existence to the creative influence of Indian ideas... No conquest or invasion, no forced conversion imposed them. They were adopted because people saw they were good and that they were good and that they could use them..."*

In the face of this history and this reality, it is not merely wrong but a grotesque misrepresentation to portray India as civilization that only received from outside. The truth, as every serious student knows is the exact opposite: India has always given a great deal more than she has received. Civilization as we know today would not exist without India.

Magical Squares

May I introduce to you Jain he is an absolute wizard at displaying the real ability of our minds to do complex mathematics simply! Yes you did hear right! Jain revels to you simple methods to do all the complex maths you can think of with only knowing the 2x, 3x, 4x, and 5x times tables.

At the same time he shows you how the world as we know it is more than what we think and can for some people bring out major shifts in your way of thinking about the world! I personally am amazed at studying the sacred geometry and what it shows about nature.

For more of an idea about what sacred geometry is then think of a crop circle and its wonderful display of precise maths and geometry.

If you feel like some enlightenment and a great relief that you can really learn maths and actually do it in your head without working it out on paper then please get along to Jain's Lecture...

Explore and discover Geometry, Art and The Mathematics of Nature in a new and challenging way with Sacred Geometry. You will thoroughly enjoy drawing 'Magic Square Mandalas' as a creative and centering process. This is a course for anyone interested in MultiCultural Art, Design, Architecture, Healing and Mathematics.

The Life of Pythagoras will be covered as well as 'The Platonic Solids'. 'The Divine Proportion', 'The Fibonacci Sequence', 'Galactic Mathematics' and related topics like, 'The Mystical Squaring of the Circle', 'The Vesica Pisces' and 'Hypercubes'/'Fourth Dimension Cubes'.

Magic Squares Lecture

During this Lecture Jain will focus our attention upon the ancient Magic Square of 3 × 3 (also known as the "Lo-Shu" in ancient China, C17th BC) which is also the centre of the Tibetan Calendar/Cosmology. It is a timeless mathematical harmonic whose sums of the columns, rows and diagonals all add up to and vibrate to 15. Symbolically the Magic Square creates Order amidst Chaos, and Equality in All Dimensions.

He has authored 4 books on this subject which translates Numbers into Art, by merely connecting the first number (1) to the last number (9). This is the Path of Least Resistance. Exquisite Atomic Art patterns are created that are inherently encoded with much Ancient Knowledge e.g.:

Numerologically, these Magic Sums of 15, 34, 65, 175 and 260, etc., equate with the many names of the Gods and Goddesses in Qaballahistic lore.

6 1 8
7 5 3
2 9 4

Vedic Mathematics Lecture:

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Do your children really understand the fragmented and confusing mathematical data taught to them? Would you be interested in another method that boosted their confidence, increased their memory skills and is presented in a fun and exciting manner?

After this course of study, you will never multiply, divide or add in the same manner that you are currently using to solve arithmetic problems like 98 × 97, etc. And no more Times Tables!

An understanding of the 16 basic Sutras or "Word Formulae" solves all known mathematical problems by using an advanced and highly intelligent system of MENTAL, ONE LINE ARITHMETIC used by the Hindus 2,000 years.

This system was reconstructed by Bharati Krsna Tirthaji (1884-1960, aka Shankarcharya of Puri) who was invited by Paramahansa Yogananda to take this knowledge, for the first time, to the western world, in 1958. The Beatle's guru: Maharishi Mahesh Yogi, who introduced Transcendental Meditation to the west, was also a student of this master and currently teaches this system in India in the spiritual Maharishi schools, whose teenage students are highly distinguished mathematical geniuses.

Learn also how the Hindu Invention of Zero (and the decimal system) led to modern technological advancements.

In this 1 and half hour Introduction you will know the basics of how to accomplish rapid calculations, mentally. No more Calculators which make the brain lazy. The brain is a mental muscle and needs to be exercised. Jain predicts that if we continue to teach children with an over-emphasis of using calculators in the class-room, over a 20 year period, the Global Brain will deteroriate.

Most children are visual, and the ability to tap into "Pattern Recognition" is the key to rapid mental calculation.

Magic Square and Vedic Mathmatics Workshop: Jain's essential work is to teach Ancient Knowledge to Children and Adults.

On Magic Squares: (Translating Numbers Into Art)

6 1 8

7 5 3

2 9 4

Imagine in front of you some playing cards and the ancient puzzle was to create a 3x3 square such that all columns, rows and diagonals that have the same sum. As shown above, the Magic Sum, aka the Magic Square Constant = 15. There is a treasure house of knowledge encoded within this harmonic matrix that the ancients knew and hid. Magic Squares for thousands of years have fascinated many cultures and many great minds like Benjamin Franklin and Albrecht Durer.

Why was it at the centre of the Tibetan Calendar/ Cosmolgy (Time Code/Applications in Time Travel)?

Why did Ambaji, of the Hindu pantheon, the ancient goddesses of Creation (the universe flows through her breast), have as her tantric symbol, this Magic Square of 3x3, which has 8 possible permutations? and which Buddha, in his enlightenment, described in detail the 8 sided nature of sub-atomic molecules: the "Acta-Kalapos". Durga and Kali and

Shakti, etc., have variations of the same. Why is it that when you translate NUMBERS INTO ART, by drawing a long connecting line, from 1 to 2 to 3, etc., to the last number, that elegant and phiratioed patterns appear? that when tiled or tessellated, produce the atomic structure of Diamond (which is verified by the clairvoyant drawings of the Occult Chemists: Charles Leadbeater and Annie Besant.

Why is it that the pattern for the Magic Square of 11x11 is almost identical to the Ley Line Grid developed by Bruce Cathy, a New Zealand pilot, which was originally the jig-saw-puzzle-map of UFO sightings in his area?

Why did Professor Searl, of the UK, have his work on levity discs and over-unity generators banned? The metals of his generators for the space-craft he built were intimately related, by proportion, to the atomic table of elements and to the opposing pairs of the above Magic Square of 3x3, e.g.: notice that all pairs around the central cell have a sum of 10, thus 2 parts gold was alloyed with 8 parts silver, or 3 parts titanium to 7 parts nickel.

Why did King Solomon the Rabbi, design his Seal based on the 3 rows of the above Magic Square being translated into a Natural Square of 3 (that has its numbers in Counting Order)?

In this day lesson, you will superimpose a Magic Square of 7x7, by rotation of 90 degrees upon itself, to reveal ancient symbols at its centre: the Hindu Peace Symbol of the Swaztika nesting within the Christian Cross!

The ancient Essenes adored the richness of symbolism within these Mystic Squares and gave accounts or Parables of Jesus (Isa or Iesu) that related to our sense of community symbolized by the magic square, that each person, so positioned, has a divine role to play, that if one number was taken away or moved, it would no longer be magical, to realise "Order amidst the Chaos", and the ancient axiom: "As Above, So Below".

About Jain

Jain first began teaching mathematics visually 20 years ago when working with dyslexic children, developing the concept of "Remedial mathematics". Not everyone learns their numbers by left-brain, rote learning, but better through visual, right-brain, the universal language of pictures and symbols. He has since authored 8 mandala, colouring-in books on Magic Squares that converts Mathematics into Atomic Art, surprising many nuclear scientists with his crystalline creations. His authorised travelling school show: "Mathemagics", is the only one of its kind and helps restore and enhance the inherent beauty in mathematics, by teaching it via theatre, with magicians and Alice-in-Wonderland type characters that appear as walking Books of Life, etc. He is currently active teaching this around Australia.

Having come from an Arabic background, the rich ornamental designs that you often see on domes and temples, are the main influences that pervades his work. He is pulling together all his vast knowledge on ancient Magic Squares, Divine Proportions (as in the mathematics of the nautilus shell), and the 3-Dimensional Geometries of the Platonic Solid Symmetries into a unique 2 hour performance on June 21st and 22nd, our Winter Equinox, the shortest day of the year. He calls this rehearsed gathering: THE THEATRE OF THE HOLY NUMBERS, where the Spirit of the 9 Numbers (like 9 Gods) incarnate into a Human Magic Square of 3x3.

Most people know of the Magic Square of 3x3 as an ancient playing-card puzzle, the task being to arrange all 9 cards into a 3x3 square in such a way that the sums of the 3 columns, 3 rows and 2 diagonals all equal 15.

This fascinating production will explore the Magic of the Magic Square of 3x3 and truly download to you all there is to know about it, in a true Keanu Reeves/Matrix-like style, just by sitting in the comfort of your chair.

If you never understood the true meaning and depth of Pythagoras' work 2,500 years ago, this mathematically-based ceremony will not only educate you but leave you enchanted.

I teach by creating the classroom as a theatre. The walls are draped with colourful Magic Square murals about 2 metres square. I wear a special MatheMagician's, magic square robe, that depicts the class lesson with the appropriate numbers and art.

Upon a large stand I have what is called a "Human Cloth Book", it has 30 pages (1,200mm square) of magic square theory and design. It is a unique hand- sewn garment that can be worn and is used throughout the whole show. One of the main lessons shows how the common Multiplication Table produces the Atomic Structure of Platinum Crystal.

I offer Creative and Multicultural Mathematics whose roots stem from Islam, India, Egypt and other cultures. I relate how mathematics can be Inter-Subjectal and may cross Key Learning Areas - Maths as Art, Maths as Science, Maths as History.

Magic Squares are ancient boxes of numbers whose sums of rows, columns and diagonals all add up to the same sum. This matrix represents Perfect Equality in all directions or dimensions. Elegant or aesthetically pleasing patterns can be created when a long, continuous line is joined in arithmetical order from 1 to 2 to 3 to 4, etc., to the last number. This is how Mathematics is transformed into Art. This line connecting such consecutive or sequential numbers is the backdoor into the world of Atomic Art or Nuclear Geometry, arriving at scientifically verified atomic structures. Order is created out of Chaos.

The Magic Square of 3x3 shown over is the centre of the Sino-Tibetan calendar/cosmology and has fascinated many cultures for millennia. The Tibetans believe that every number has a Soul, as does any insect, worm or creature and therefore a Magic Square is a holy thing compared to a Spiritual Community of Souls!

Many sacred symbols emerge when the original Magic Square Pattern or Yantram (singular of Yantra which is the plural) is rotated upon itself. Like snowflakes, millions of Magic Squares can be created. There are 8 permutations of the primal 3x3 Magic Square known also as The Lo-Shu from China 4,000 years ago. There are 880 permutations of the 4x4 Magic Square, but as we get to the 5x5s and 6x6s there are millions! Mathematics is a Universal Language. That means that it will always exist, without decay or corruption.

The concept of 1,2,3 is superior to the concept of A,B,C as English, French, German or any other language eventually changes with slang, etc., but not numbers. As Pythagoras said, in the 6th century BC, "NUMBER IS THE BOND OF ALL THINGS". This Galactic Mathematics is also called "FIXED DESIGN", i.e: it is Timeless and Constant.

Some questions to ponder upon:

* How could the Aryan civilisation, near the Hunza Valley, 3,000 years ago, mentally compute mathematical operations that today only a calculator could achieve?
* And if the maths could not be done mentally, how could they do everything we know today, IN ONE-LINE? e.g.: converting the fraction 1/19 into a decimal is an 18 digit answer that requires 35 lines of exhausting long division, how could they do it in a one simple line solution?
* Why is NASA secretly using this MENTAL ONE-LINE SYSTEM OF ARITHMETIC in the field of Advanced Artificial Intelligence? and not incorporating it into our current mathematics curriculum?
* Why are many of Bill Gate's mathematicians Indian?

- Is the decimal form of Pi (P) a sonic code for dedications to Krishna?
- What does the Bhagawad Gita have to say about this lost science of Clairvoyant Calculation that the ancient Temple Builders employed?
- How can you become a Human Bio-Calculator?
- Is the current Western Maths Curriculum keeping our Children in a state of limited intelligence?

The answer to all these questions are found by understanding the ancient system of Vedic Mathematics as rediscovered by Bharati Krsna Tirthaji (1884- 1960), of Puri Govardhan Math, dux of India in Mathematics and a Sanskrit Philosopher who wrote and spoke perfect English, he reached the position of a Shankacharya (like a Dalai Lama), renunciated several high officical positions in society to retreat into a forest for 7 years as a yogi, between 1911 and 1918 in which time he collected all the lost mathematical knowledge (Ganita Sutras), improved and reconstructed them to 16 basic Sutras or word formulae that solve every known mathematical problem. These 16 Volumes relating to the 16 Sutras was our legacy from the ancient past.

This highly intelligent and sophisticated system of Mental, One-Line Arithmetic was taught by The Master to his main student, who today is the leading exponent in the world: Maharishi Mahesh Yogi, the long white-haired and bearded yogi that many of the 60s generation know of as the Beatleis guru. He is considered as the greatest human bio-calculator on the planet today, holding the vision of raising the collective consciousness of all nations by re-introducing the lost Vedic Mathematics Sutras early in a childis spiritual and educational development. Imagine a maths class in India today , like in the times of old when knowlege was passed on as an oral tradition, and the teacher quickly calls or drills out certain numbers to multiply or divide and the children instantly reply with the correct intuitive answers! Today, in schools where Maharishiis Transcendental Mediatation technique are used students perform remarkably well academically, viewing Mathematics as a Universal Language and therefore often winning the highest competitve prizes in mathematics.

Maharishi organised the scattered Vedic Literature into a complete science of consciousness for both its theoretical and practical value, describing the 16 Sutras as fundamental impulses of intelligence that structure all the Laws of Nature and those that govern the Universe.

Maharishi, the Founder of TM, once said: "Vedic Mathematics is that one field of knowledge which fulfils the purpose of education by developing the total creative genius of the individual, giving him/her the ability to be always spontaneously right and automatically precise so that his/her action, supported by Natural Law, is always effortlessly fulfilling."

Unfortunately, in 1958 after Bharati Krsna Tirthaji returned from the USA tour, sponsored by Paramahansa Yogananda (who introduced Yoga to the West and created the Self-Realisation Foundation) all of Bharati Krsnais 16 Volumes for the 16 Sutras were confirmed lost. Disciples observed that people in the streets of India were imitating Tirthajiis lectureis and dazzling audiences with mathematically misused pschic powers, and begged the Master to write a book just before his death in 1960. This single volume, called VEDIC MATHEMATICS, has become the bible on this subject and spawned a thousand web-sites worldwide. Maharishi schools disseminated the teachings to England where certain schools have adopted the Vedic curriculum but the irony is that today India is struggling to release the British curriculum and NASA have adopted it fully in the realms of advanced robotics. Calculations that can be solved as quick as lightning are a great tool to adopt, but you wouldn't want to teach it worldwide in the fear that you may churn out a generation of child geniuses that may threaten the intellectual status quo.

The gift that the Hindus gave to world, thousands of years ago, and that which is currently responsible for a global silicon chip technology, was none other than the invention of ZERO. We call our common numbers Arabic Numerals but really they extend back to the Hindu concept of creation and void known as Bindui or The Zero Point. In Maths, it makes the magic work e.g.: if you wish to multiply 98 × 98 we need to understand Zero and Unity. All Vedic Maths is based on the understanding of Unity Consciousness which means they utilise processes or Number Bases that correspond to: 0, 10, 100, 1000, 10000, etc., all of which add to 1.

Sacred geometry is a fascinating and complex field of study. Possibly the most commonly known aspect of Sacred Geometry is, The Golden Mean, sometimes referred to as the The Sacred Cut or The Phi Ratio. This ratio has connections with Galactic and Atomic mathematics, as revealed in ancient temples and pyramids; in classical paintings like the Mona Lisa; in the cross-sectional view of the D.N.A. molecule; in the nesting of the 5 Platonic Solids; its occurrence encrypted in the Fibonacci Sequence: 1,1,2,3,5,8,13,21,34,55,89,144, etc.; in the counter-rotating fields or spirals of the sunflower and pine cones.

This is the study of the Universe as found in all parts of the Human Body. This is the mathematics of infinity; Phi expressed as 1:1.618033, etc. (The decimal part goes on forever without any detectable repetitious pattern, like pi).

This Transcendental Number has fascinated many cultures and individuals for millennia. This Phi Ratio is part of our Ascension Process and is an important clue to accessing or Multi-Dimensionality.

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Sacred geometry is a fascinating and complex field of study. Possibly the most commonly known aspect of Sacred Geometry is The Golden Mean, sometimes referred to as the The Sacred Cut or The Phi Ratio. This ratio has connections with Quadratic and Algebraic mathematics, as revealed in ancient temples and pyramids; in classical paintings like the Mona Lisa; in the cross sectional view of the DNA molecule; in the nesting of the 5 Platonic Solids; its occurrence observed in the Fibonacci Sequence, 1,1,2,3,5,8,13,21,34,55,89,144, etc.; in the counter-rotating fields of spirals of the sunflower and pine cones.

This is the study of the Universe as found in all parts of the Human Body. This is the mathematical ratio of Phi and expressed as 1:1.61803, etc. This decimal carries on and on forever without any detectable repetitive pattern, like pi.

This Transcendental Number has intrigued many cultures and individuals for millennia. This PHI Ratio is part of our Ancient Heritage and is an integral part of Vedic Math Fundamentally.

2

Vedic Mathematics : As Relevant Today as Ever

By investigating the knowledge and viewpoints in the many topics found in Vedic culture we can certainly see that the practice and utilization of this Vedic knowledge can indeed assist us in many ways. In regard to all the trouble we presently find in this world, maybe it is time to look at things through a different and deeper view to find the answers and directions that are so needed. The knowledge and understandings of this great Vedic culture may indeed be what will help us see through the fog of confusion that seems to envelope so much of society.

What we find in Vedic culture are areas of study, progress and expression that are as relevant today for human advancement as they were hundreds or thousands of years ago. India and its Vedic culture has contributed much to the world, such as its music, beautiful forms of art and architecture, martial arts, astronomy, holistic medicine in Ayurveda, and the mathematical system based on the number ten, along with its yoga and philosophy.

In the United States, yoga has exploded into a three billion dollar industry. A recent survey (at the time of this writing in 2005) showed that 16.5 million people are practicing yoga, or 7.5 percent of the United States. Also, the Yoga Journal magazine has grown from a circulation of 90,000 in 1998, to 170,000 in 2000, to 325,000 in 2005.

Vedic mathematics is another example of its contribution to world progress. It is an ancient development that continues to play an important part in modern society. Without the advancements in math that had been established by Vedic culture as far back as 2500 BC and passed along to others, such as the Greeks and Romans, we would not have many of the developments and inventions that we enjoy today. The Greek alphabet, for example, was a great hindrance to calculating.

The Egyptians also did not have a numerical system suitable for large calculations. For the number 986 they had to use 23 symbols. Even after the Greeks, the Romans also were in want of a system of mathematical calculations. Only after they adopted the Indian system that was called Arabic numerals did they find what they needed. Weights and measures and scales with decimal divisions had been found from that period which were quite accurate.

The difference was that Vedic mathematics had developed the system of tens, hundreds, thousands, etc., and the basis of carrying the remainder of one column of numbers over to the next. This made for easy calculations of large numbers that was nearly impossible in other systems, as found with the Greeks, Romans, Egyptians and even Chinese. The Vedic system had also invented the zero, which has been called one of the greatest developments in the history of mathematics.

The numeral script from India is said to have evolved from the Brahmi numerals. This spread to Arabia through traders and merchants, and from there up into Europe and elsewhere. It became known as the Arabic numerals, yet the Arabians had called them "Indian figures" (Al-Arqan-Al-Hindu) and the system of math was known as hindisat, or the Indian art.

Vedic culture already had an established mathematical system that had been recorded in the Shulba Sutras. These are known to date back to the 8th century BC. The name Shulba Sutras meant "codes of rope". This was because such calculations were used for measuring precise distances for altars and temple structures by using lengths of rope.

The Shulba Sutras were actually a portion of a larger text on mathematics known as the Kalpa Sutras. These and the Vedic mathematicians were recognized for their developments in arithmetic and algebra. Indians were the first to use letters of the alphabet to represent unknowns. But they were especially known for what they could do in geometry. In fact, geometrical instruments had been found in the Indus Valley dating back to 2500 BC. Furthermore, what became known as the Pythagorean theorem was already existing in the Baudhayana, the earliest of the Shulba Sutras before the 8th century BC. This was presented by Pythagoras around 540 BC after he discovered it in his travels to India. So this shows the advanced nature of the Vedic civilization.

After the Shulba Sutras, Vedic mathematics enjoyed further development in the field of Jyotish, Vedic astronomy, which used all forms of math. Indian mathematicians continued creating systems that were not known in Europe until much later in the Renaissance period. For example, Aryabhatta in the 5th century introduced sines and versed sines, and is credited as the inventor of algebra. He is said to be the first to state that the Earth travels around the sun. However, the ancient Vedic texts have described this many years earlier, which shows the wisdom of the early Vedic seers.

Aryabhatta was followed by Brahmagupta (7th century) who was the great mathematician that especially developed the use of zero and was the first to use algebra to solve problems in astronomy. Next was Mahavira (9th century) who made great strides in the use of fractions and figuring out how to divide one fraction by another. Then there was Bhaskara (12th century) who made progress in spherical trigonometry and principles of calculus before Newton by 500 years. He used it to determine the daily motion of planets.

The Vedic system of math, as explained in the sutras, also reduced the number of steps in calculations to merely a few that otherwise required many steps by conventional methods. Thus, this ancient science is still worthy of study today.

A well-developed medical system was in existence by the 1st century AD. Progress in medicine led to developments in chemistry and the production of medicine, alkaline substances and glass. Colourfast dies and paints were developed to remain in good condition over the centuries. The paintings in the caves of Ajanta are a testimony to this.

Vedic art is another ancient development that still holds much appreciation in modern times. Art in the Vedic tradition was never a mere representation of an artist's imagination. It was always a vehicle to convey higher truths and principles, levels of reality that may exist beyond our sense perception. It was always used to bring us to a higher purpose of existence and awareness. In this way, it was always sacred and beheld the sacred. Still today it is used to allow others to enter into a transcendental experience. It may also present the devotional objects of our meditation.

Vedic paintings or symbols are unique in that they can deliver the same spiritual energy, vibration and insight that it represents. In other words, through the meditation and devotional mood of the artist, the art becomes a manifestation of the higher reality. In this way, the painting or symbol becomes the doorway to the spiritual essence contained within. They are like windows into the spiritual world. Through that window we can have the experience of darshan of the Divine or divinities, God or His associates. Darshan is not merely seeing the Divine but it is also entering into the exchange of seeing and being seen by the Divine.

Thus the art, or the Deity, is beyond mundane principles or ingredients, such as paint, paper, stone or metal with which it may be made, but it becomes completely spiritual through which the Deity can reveal Himself or Herself. Thus, the truth of spiritual reality can pierce through the darkness of the material energy and enter our mind and illuminate our consciousness.

To convey higher realities in paintings and sculpture, everything has a meaning. The postures, gestures, colours, instruments or weapons, everything conveys a principle or purpose, which often must be explained to those who lack understanding. Thus, knowing the inner meaning of the painting increases its depth for those who can perceive it, which makes it worthy of further meditation and contemplation.

As with art, dance in India was not merely an expression of an artist's emotional mindset or imagination, but was meant to be an interpretation or conveyance of higher spiritual principles or pastimes of the Divine. In fact, in the Vedic pantheon Shiva is known as Nataraja, the king of dancers. Shiva's dance was also not without a more significant purpose. His dance was based on the rhythm of cosmic energy that pervades the universe, and the destruction of the illusory energy by which all souls are given the opportunity for release from the illusion to attain liberation, moksha.

In this way, traditional Indian dance is highly spiritual and often accompanies important religious rituals and holy days and festivals. Vedic dance goes back to prehistoric times. Bharata Muni wrote his Natya Shastra, science of drama and dance, over 2000 years ago. In it he explains that it was Lord Brahma, the secondary engineer of the universal creation, who brought dance (natya) and drama to the people of Earth millions of years ago, shortly after the Earth was created.

Now dance has evolved into a tradition involving various schools and styles but with strict discipline. It is not uncommon that Indian families will have their daughters spend at least several years or more in such study and practice. There is a precise method of postures, facial and hand gestures (mudras), and movements, along with footwork that must be learned and synchronized to the beat and music in order to convey specific meanings, moods and stories to the audience. Many temples, especially in South India, were known for maintaining large groups of dancers that performed at festivals and religious functions.

When the dance is performed according to the spiritual standards, which some view as similar to the practice of yoga, even the dancers can invoke a high degree of spirituality in their own consciousness and bring unity between their inner selves and God. Then the transcendental atmosphere can manifest and draw the Divine to appear in the performers on stage. Thus, the environment becomes transformed and the audience may also experience darshan of the Divine and experience an inspiring upliftment in their own consciousness. In this way, the dance is divine beauty in motion. Or it is a way of invoking the spiritual dimension into our midst. Few other forms of dance attempt to do this.

Various schools of dance include Bharata Natyam, Kathakali, Manipuri, Orissi, Kathak, Mohini Atam, Krishna Atam, Bhagavata Mela, etc. Thus, we may have many dances that convey stories from the Ramayana and Mahabharata, or Krishna-lila from the Bhagavata Purana. Nowadays this ancient art of Indian dance is enjoying a wide audience and a prominent place on the international stage.

So, as we can see, Vedic culture and its many areas of knowledge and devotional expression are still as relevant today as it was thousands of years ago. And humanity can benefit from it by introspection and in spiritual as well as material development as it did in the past.

The power of the Dharma and the relevancy of Vedic culture are found in the number of tools it has always provided in order for humanity to reach its fullest potentials, both as individuals who are searching for their own fulfilment and spiritual awakening, and as a society that can function in harmony with nature and cooperation amongst themselves.

By investigating the knowledge and viewpoints in the many topics found in the Vedic tradition we can certainly see that the practice and utilization of this Vedic knowledge can indeed assist us in many ways. Let us take a look at a few.

Ayurveda

Ayurveda is the Vedic system of holistic medicine. It has become quite popular in the West and is continuing to gain ground and acceptance. To understand briefly what Ayurveda is, I let Pratichi Mathur, an Ayurvedic practitioner herself, tell us about it from the book, "Vedic Culture: The Difference It Can Make in Your Life":

"So what is Ayurveda exactly? Literally translated from Sanskrit it is composed of two words 'Ayus' which means life and 'Veda' which denotes knowledge. So Ayurveda is the knowledge of healthy living and is confined not only to the treatment of diseases. Life is a vast, and an all-encompassing phenomena, which includes death. On one end, life is a celebration of birth, growth, child bearing, youth and sexuality; on the other end, life also brings forth disease, decay, aging, and loss of vigour. Ayurveda is that ancient art and science that helps us understand this very 'life' with all its different shades and colours; understand how best we can undertake this journey; and how we transition through its different phases, example from teenage, to adulthood, to maturity, etc. Following the principles of Ayurveda brings about a profound understanding of the inner ability to have sound body, mind and spirit. From this point of view, Ayurveda is a compendium of life and not disease. This is a major agenda indeed for any system of medicine, but can it be any less--especially if true healing has to take place. Perhaps, this is exactly why Ayurveda manages to get to the root of the disease that distresses the mind or the emotion that ails the body.

"Ayurveda has twin objectives--maintaining the health of the healthy, and cure illnesses of the diseased. Ayurveda, which is not just a system of disease and its management, but literally a living dynamic philosophy and manual on the art of living, is well fitted to meet its objectives. On one hand Ayurveda offers treatments like Panchakarma or even surgery for the diseased; and on the other hand Ayurveda offers preventative medicine for the healthy. These include elaborate details for following ideal daily and seasonal routines, specialized diets for optimizing health and immunity (Ojas), Rasayana Chikitsa (promotive therapy), Vajikarna Chikitsa (aphrodisiac therapy), Swasthavritta (regimen to stay healthy furnishing details on topics such as exercise, smoking for health), Sadachar (social hygiene), etc.

"Ayurveda advocates a complete promotive, preventive and curative system of medicine and includes eight major clinical specialties of medicine namely,

(1) Medicine (Kayachikitsa),
(2) Surgery (Salya Tantra),
(3) ENT (Salakya Tantra),
(4) Pediatrics (Kaumatabhritya),
(5) Psychiatry (Bhutvidya),
(6) Toxicology (Agad Tantra),
(7) Nutrition, rejuvenation and geriatrics (Rasayan tantra), and
(8) Sexology and virilization (Vajikarana).

This shows what a developed science Ayurveda was in ancient times.

"The exact origin of Ayurveda is lost in the mists of antiquity. Since Panini is placed at 7th century BC and Ayurveda depicts non-Paninian Sanskrit grammar, it is logical to place Ayurveda between 6th -10th Century BC. Tracing the continuity of Ayurveda, it is natural to look for the continuing thread in India's ancient Vedic tradition. Although the term Ayurveda, does not seem to appear in the Vedas, and it appears first in Panini's Ashtadhayayi, however, there are positive evidences to show that in the Vedic period, medicine as a profession was prevalent. The Rig Veda and the Atharva Veda both mention that there were thousands of medical practitioners and thousands of medicines. References to Ayurveda are found as early as the Rig Veda. The three Rig Vedic gods Indra, Agni and Soma relate to the three biological humours: Vata, Pitta and Kapha. References are made of organ transplants as in the case of the artificial limb of queen Vishpala, daughter of King Khela. The functions of physicians are also described in the Rig Veda.

"Rishi Sushruta, famous Ayurvedic Surgeon, also holds that Ayurveda is a supplement (upanga) of the Atharva Veda. While several other sources including the famous Hindu epic Mahabharata speak of Ayurveda as an upanga of Atharva Veda; several other schools of thought hold Ayurveda as a fifth Veda (Panchamveda). Perhaps Ayurveda grew from Atharva Veda first as a branch and then as a comprehensive vast system deserving it's own status, or it developed parallel to the four Vedas as an independent knowledge (with close resemblance to the Atharva Veda)."

Jyotish—Vedic Astrology

Jyotish is the Vedic form of astrology, which is an ancient science and is also being accepted and gaining popularity in the West. Vedic Astrology is meant to help the individual better find his or her way through life. It is to assist in discovering one's highest proclivities, personality, character, qualities and traits and what may be one's best direction for a career, and other things. Thus a person will least likely waste one's time in unfulfilling activities, professions or pursuits.

To further our understanding of Jyotish, I let Chakrapani Ullal, one of the most well-known Vedic Astrologers, describe it as taken from the book, "Vedic Culture: The Difference It Can Make in Your Life":

We turn our attention now to the subject of a branch of the Vedas called Vedic astrology or Jyotish, which is called the 'eye of the Vedas'. It has a cognizing influence of the truth of life and self-knowledge. It acts as a mirror to an individual without which one may not know how to approach life most effectively. It is also called the 'Science of Time'. Time is the source power that rules the universe. All things originate through the procession of time. Hence, Vedic Astrology constitutes the science that maps the structure of time. Astrology is considered divine knowledge that is pure, supreme, secret, and exalted.

Astrology can be defined as the science of correlations of astronomical facts with terrestrial events, and demonstrates the Vedic understanding of the universal interconnectedness and interdependence of all phenomenon, that microcosm and macrocosm are but reflections of one another. Just as mathematics is the organizing principle of science when dealing with inanimate matter, so also astrology is the organizing principle which deals with life and its significance in relation to all living bodies. The planets are seen as reflectors or transmitters of light and solar energy. The solar and planetary rays, like radio waves, affect biological and psychological processes. The rays of influence are unseen vibrations that are not perceptible to the physical eye.

Astrology gives insight and guidance to the fortunes and misfortunes of men, issues of empires and republics, floods and earthquakes, volcanic eruptions, plagues, pestilence and other incidents concerning terrestrial phenomena in relation to the regular movements of the planets.

Over 10,000 years ago the ancient sages, in their superconscious state, cognized that there is energy in planets, and that they send out different rays at different angles which bear influence on everything animate and inanimate on other planets. Through their sensitized intuition and repeated observations these highly evolved souls were able to find out the different characteristics inborn in the planets and also discovered that each rules a distinctive part of the human mind/body. It was also found that particular groups of stars known as constellations have different characteristics, and that they modulate the influence of the planets.

Astrologers say that there are two forces, Daiva and Purushakara, fate and individual energy. The individual energy can modify and even frustrate fate. Moreover, the stars often indicate several fate possibilities; for example, that one may die in mid-age, but that if, through determination, one gives attention in that area it can be overcome, one can live to a predictable old age. Thus, astrology does not say that events must and should happen, but gives the benefic and malefic tendencies which can be directed or modified through conscious effort. The horoscope shows a man's character and temperament. Though it may show that he could become a criminal, it does not mean he is fated to become so. What it means is that he is just the sort of person who will have criminal tendencies, but they can be checked by proper care and training. Additionally, if emotional and financial challenges are indicated in any particular year, one can certainly meet the crisis better if one knows that it might occur.

"Then, how would one define astrology? It is the philosophy of discovering and analyzing past impulses and future actions of both individuals and nations in the light of planetary configurations. Astrology explains life's reactions to planetary vibrations."

Vedic Gemology

Gemology is an important field in today's market. But when we speak of Vedic gemology, we do not mean that it is merely for judging the value of a gem. The Vedic purpose in gemology is to determine the best type of quality gem for a person to wear. Thus, Vedic gemology worked in conjunction with Ayurveda and Jyotish to establish the best gem a person should wear for health and positive influence. To give a little more understanding about this increasingly recognized field, I include the following description by Howard Beckman, a qualified and practicing Vedic gemologist, from the book, "Vedic Culture: The Difference It Can Make in Your Life":

"It is a field that is making great strides medically by using gems for illness and disease both of the physical body and the mind. It is a noninvasive therapy that has produced definite repeatable results medically. (It should be noted that only natural gems, not synthetic, have this inherent energy and also that certain gem treatments commonly used for colour or clarity enhancement will render the gem 'dead' and ineffective.)

Our research and record keeping of case histories of gem use in jewellery for astrological reasons has also allowed us to not only prove the efficacy of gems, but in "debunking" commonly held incorrect notions as far as how to recommend them, as well as baseless superstitions.

"It is the energy force of the cosmos that sustains all living organisms. This energy is called 'prana'. It energizes our bodies throughout life until it leaves at the time of death, leaving the gross material body to decay and return to the elements from which it arose. The Vedic scriptures calculate our life spans in the number of breaths we are allotted during our lives.

If we use this energy more quickly, then the life span will be shorter. (Long distance runners are renowned for dying in their 50's.) If we conserve our energy, especially through systems such as the yoga system, then the life span may be extended. The Ayurvedic system of healing first evaluates the intake and distribution of prana within both the physical and subtle (ethereal) bodies of an individual.

"Gem therapy has been used by many ancient cultures and especially the wearing of gemstones on the body had great significance for the Vedic culture, other than the purely cosmetic or ornamental value that gems are mostly used for today. The science of Ayurveda when combined with Vedic astrology gives a wealth of knowledge in the correct application of gemstones to amplify planetary rays, which can have a dynamic effect on one's physical and emotional health, one's ability to prosper materially, and the general well-being of individual persons here on earth.

"As Gems have such vibratory qualities, we may utilize them to not only affect the brain, but also the higher vibrations in the physical body necessary for healthy functioning of all our internal and external organs. Dr. Young and Bruce Tainio of Cheny University in Washington have made the following statements from their research in this regard. 'The average frequency of the human body during the daytime is between 62 and 68 cycles per second. If it drops below this rate the immune defense system will start to shut down. Cold symptoms appear at 58 cycles, flu at 57, candida at 55, glandular fever at 52, and cancer at 42 cycles per second'.

"Natural (meaning from the earth, which does not include synthetic, man-made material), untreated gemstones, which are repositories of cosmic colours, can restore the pranic energy to the cells of the body, so that its natural vibratory rate and normal health may be regained when it is in a diseased condition. Blue sapphire can tranquilize or have a sedative effect. Emerald can be used as an analgesic. Yellow sapphire has antiseptic properties, and diamond's ability to stimulate cell growth are just a few examples of how gems can affect the healing process in the body."

Vastu Shastra

Vãstu is the Vedic science of architectural and home arrangement. It made its way through the orient and became known as Feng Shui, which has made particular progress in popularity in the West. However, Vastu is a particular science that deals with the flow of energy through a house or building for the highest benefits. It is not enough to merely arrange a house so it looks nice or that there is a good flow of energy through it. But there is much that depends on the directions in which things are facing or which parts of the building in which certain activities are performed.

To get a little more insight into the Vedic science of Vastu, I have included the following description by Arun Naik, an architect that practices the science and art of Vastu Shastra. Again, this is taken from the book, "Vedic Culture: The Difference It Can Make in Your Life":

The Vedic and the Agamic traditions of ancient India always held that the microcosm is a reflection of the macrocosm. A dwelling is an ecological unit, a microcosm which reflects the Cosmos, the macrocosm. Vastu Shastra is the applied aspect of this philosophy, a highly refined method of creating a living space which is a miniature replica of the cosmos as perceived by the Vedas. Vastu Shastra is about emulating the attributes of the Cosmic Space, about bringing the divine sentinels of Cosmic Directions into our homes, about creating Harmony by creating a living environment where the forces of nature are balanced and at peace with each other.

Sri Aurobindo has said... 'Indian sacred architecture of whatever date, style or dedication goes back to something timelessly ancient and now outside India almost wholly lost, something which belongs to the past, and yet it goes forward too, though this the rationalistic mind will not easily admit, to something which will return upon us and is already beginning to return, something which belongs to the future.' (SA, The Renaissance in India)

There is a prayer is Sama Veda:

> *May there be peace in the sky, may there be peace in mid region, may there be peace on earth, may there be peace in the waters, may the medicinal plants be peaceful, may the forest be peaceful, may there be peace in gods, may Brahma be peaceful, may all the creation be peaceful, may there be peace and peace only, may such peace come to us.*

"Vastu is about creating an Inner Space, the chidakash, where this divine peace can park itself. And it achieves it by creating a harmonious external environment—the bahyakash.

At a more earthly level, Vastu Shastra aims at establishing a dynamic balance between Form and Energy so that harmonious conditions are created for the inhabitants. Vastu buildings have harmonious energies and they promote stability, prosperity, happiness, and mental peace for the occupants and owners.

The principle of Vastu is that the Cosmic World with its order and stern discipline has been built by the gods who occupy all the spaces, from the celestial Space within the Cosmic World to the little spaces in our homes, and even our mental space, chidambaram. Man's existence in the Cosmic World has a purpose: it must ascend to immortality and godhood; and the gods, having occupied man's inner Space, strive to create different states in man's consciousness for his ascension from mortality and low nature to Truth, godhood and immortality.

Vastu Shastra helps the effort of the gods by creating an external space—a dwelling, a place to worship and meditate, or a place to work by applying the same laws which the gods have used to create the Cosmic World. This, indeed, is the ultimate function and the highest objective of Vastu Shastra.

So here we can see how various aspects of the ancient Vedic culture are still applicable today and can provide assistance in our attempts to reach our highest potential, both materially and spiritually. This is the constant and higher nature of the power of the dharma that can be recognized and utilized generation after generation.

Vedic Mathematics

Jagadguru swami, Bharathi Krishna Tirthaji maharaja established the Vedic mathematics system from the ancient Vedic texts coupled with natural process of mathematical reasoning.

It is based on sixteen sutras which are in the form of short rules of working or aphorisms or formulae. Sutras provide rules for special cases and this envisages continuous practice of their application. A sutra is a thread of knowledge, a theorem, a ground norm, a repository of proof. A single sutra would generally encompass a wide and varied range of particular application.

Veda has the derivational meaning, the fountain head and unlimited store house of all knowledge. The Vedas are well known as four in number Rig, Yajur, Sama, and Adharvana, but they have also the four Upa Vedas and the six Vedangas all of which form an indivisible corpus of divine knowledge.

Upa Veda

Ayurveda

Gandharva Veda

Dhanur Veda

Sthapatya Veda

Veda

Rig Veda

Sama Veda

Yajur Veda

Adharvana Veda

To our surprise, the Vedic Maths principles could very well be fit in multiplication and division and further to fractions, squares and square roots in abacus system for any calculation.

Multiplication: Conventionally in abacus, multiplication is done number by number in 1st round and 2nd round multiplication and so on. But this is cumbrous with large number of digits. Here come to play are the vedic sutras viz all nine and last from ten and vertically and cross wise. This help to get in one line answer.

Division: In abacus, division is done with assumed quotient on trial basis for big divisors. The different vedic methods viz straight all from 9 and last from 10. Transpose and adjust proportional and straight division could be easier with small or big numbers.

Maiden Attempt: To the few of the author infusing Vedic Maths Sutras in abacus system, may the first of its kind the world over. And we, the Indians should be proud of mastering Indian Vedic Maths and oriental abacus system in one stroke.

Ancient Indian Mathematics

"I have never found one among them who could deny that a single shelf of a good European library was

worth the whole native literature of India and Arabia." —Thomas Babington Macaulay, Minute on Indian Education, 1835.

India was usually known to outsiders until very recently as the land of naked sadhus, snake charmers, overtly religious mystical land which taught the world how to make love. Such a perspective definitely helped the colonisers in assigning an inferior status for the colonised and justify their acts of colonisation and "civilisation". Macaulay's remarks in his Minute on Indian Education is a good read to understand the British perspective then about India. Many influential political leaders (Gandhi for example) who had renounced the more superstitious religious beliefs, in favour of less superstitious beliefs also contributed to the mainstream western perspective on India remaining a status quo. This is surprising considering the fact that even during Gandhi's time, there were huge difference of opinion between Gandhi and other more rational minded people like Tagore and Ambedkar.

Gandhi in his essay Hind Swaraj, strongly condemds technology and industrialization, whereas Tagore believed in science, industry and rationality. Tagore was particularly disgusted by Gandhi remark that the 1934 Bihar earthquake was a divine retribution for ill treatment of dalits. Tagore, who was as much against caste exploitation as Gandhi was, however could not stand such an irrational explanation for the earthquake suggested by Gandhi. Their debates on this and many other issues (like Gandhi's belief that everyong should use the carka for atleast 30 mins a day) in the form of letters make a very good read.

Another non ignorable figure who had serious issues with Gandhi's irrationality was Ambedkar. The latter firmly believed in the evils of caste system, where as Gandhi who was against the opression against dalits, nevertheless believed in the sanctitty of the age old caste system and even justified its working as a means to achieving social stability.

You can read Ambedkars response to Gandhi's views on caste in his essay "Annihilation of Caste". It looks like Tagore and Ambedkar were not at all influential in changing outsiders outlook towards India. Gandhi definitely takes the spotlight.

Gladly enough, thanks to the IT age, outsourcing and the recent economic boom, this image is rapidly changing. Such a categorization, not only misrepresented India, but also ignored completely many other aspects like agnosticism (Buddhism), atheism (Carvaka) and science and mathematics which were very much being developed in ancient India. Buddhism, which originated in India and spread as a rebel against caste system and rituals, was in full glory for about a millenium and was one of the prime exports of India during those days to China and other eastern countries. So much that China refered to India as the Kingdom of Buddhism. Rationalism and skepticism too, was a school of thought which cannot be ignored during these times. Hinduism, Buddhism, Jainism, Atheism and many other schools were competing at a time for acceptance as a mainstream philosophy. Amartya Sen's book The Argumentatvie India forms an excellent read on this subject.

As far as ancient Indian sciences are concerned, it was developed and transmitted to regions outside India by Chinese and Arabs. Many mathematics and science texts were translated by Al-Beruni, Al-Khwarizmi (who is responsible for the temrs algebra and algorithm) into Arabic and found its way to Europe there on. One of the prime motivations for doing mathematics those days was astronomy. Approximations for trigonometric values, pi, circumference of circle, etc., were developed. Bhaskara's approximation for sin(x) as a rational function of x, Brahmagupta's calculations of eclipses, Aryabhatta I's approximation of pi, Aryabhatta I's method to solve linear Diophantine equations are just some examples amongst a huge literature.

Broadly, ancient Indian mathematics can be categorized into the following based on their period of development.

- Pre Vedic Indus mathematics dating back to as early as 3000BC. Development of mathematics were highly influenced by practical applications like measuring scales, calculating brick ratios, etc.
- Vedic or Sulbasutras, which contained rules to construct altars for various rites and rituals. Various constructions based on Pythagoras theorem are listed, approximation to square root of 2, approximation to pi are some of the achievements. This period probably lasted till around 500BC.
- Jaina mathematics, from 600BC to 500AD. Prime achievements are various notions of infinities, Pascal's triangle, form of set theory, operations with roots of order larger than 2, etc.
- A set of manuscripts were found around 1880s, called the Bakhshali manuscript. The dates of these manuscripts are assumed to be around 400AD. This book contains sets of problems and solutions in linear equations, fractions, square roots, etc.
- Golden age of Indian mathematics set off by Aryabhatta I. Numbers start becoming more abstract and makes it possible to consider zero and negative numbers. Brahmagupta, Mahavira and Bhaskara lists down rules for multiplying, subtracting and (wrongly) dividing by zero. All of them missed that dividing by zero does not make sense :) It was during this period that Al-Khwarizmi, who lived from 790 to 840, wrote Al'Khwarizmi on the Hindu Art of Reckoning and passed on Indian achievements to Islamic and Arabic countries. Ibn Ezra, who lived from 1092 to 1167 in Spain also helped transmit Indian number system to Europe.

- *Keralese Mathematics:* The golden age started to decline after Bhaskara's works around 1200. There is some consensus that political instabilities are partly to be blamed for this decline. Despite all these, mathematics continued to develop in Kerala. The time period between 1400 and 1600 is considered to be the peak period of this development. In addition to building upon and extending previous works by Bhaskara, Brahmagupta, Aryabhatta, etc., one of the significant achievements of this period was mathematical inductive proof in the works of Nilakantha, Jyeshtadeva. The prominent figure during this time period is definitely Madhava of Sangamagramma. He built tools essential for modern analysis. Though not fully certain, he is supposed to have independently derived taylor series expansion for arctan, sin, cos and many other function. A list of 13 different expansions attributed to Madhava is listed here. Madhava also managed to calculate the value of pi to 17 decimal places (3.14155265358979324), much ahead of his contemporaries.

People have started recognizing and acknowledging role of Indians, but awareness still seems to be a big issue. I think it will take some time before terms like Madhava-Gregory series, Leibniz-Gregory-Madhava constant, Hemachandra-Fibonacci series, Bhaskara-Brouncker Algorithm, Aryabhatta algorithm, Aryabhatta Remainder Theorem, Aryabhatta Kuttaka, etc., become commonplace.You might want to check the following links to dig deeper into the history of Indian mathematics.

The Origin of Zero

The first thing to say about zero is that there are two uses of zero which are both extremely important but are somewhat different. One use is as an empty place indicator in our place-value number system. The second use of zero is as a number itself in the form we use it as 0 (or the absence of something).

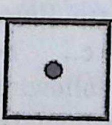
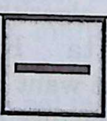
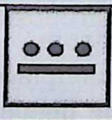
The use of zero as a numeral appeared sporadically in Egyptian number systems. It was used, however, only between two numbers to indicate an empty position, never at the end of a number. The Mayans also had a zero symbol. Around 650AD the use of zero as a number came into Indian mathematics and the first use of zero as a place holder in positional base notation is considered by some to be due to Arab mathematicians.

The 'Base' System for Numbers

General modern mathematics uses a base 10 numbering system e.g. 1,2,3....10,11. Other systems exist such as digital computation using binary, base 2. A useful site on base numbering systems can be found here.

However, in early times before written numbers it is likely that numbering systems used a base 5, probably because the hand has five digits.

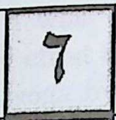
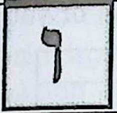
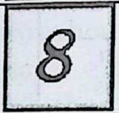
The use of 20 as a grouping number was used by many cultures throughout our earth's history. This was because people have twenty digits (fingers and toes). The Mayan culture developed a rigorous numbering system that was consistent with place values being multiples of 20. The Mayans used a system of a dot for one and a line for 5.

		
1	5	8

The Sumerian culture developed a numbering system related to 60 around 5000 years ago. Each place value to the left is equal to 60 times the place value to the right. e.g. 3600, 60, 1, 1/60, 1/3600.

Number Sytems in Different Languages

Many languages have their own mathematical numerals. However, some of these numerals have a passing and sometimes confusing resemblance to numerals as used in European languages.

	'Arabic' numeral for the number '6'
	Hebrew numeral for the number '6'
	Bengali numeral for the number '4'

Vedic Sutra—All from 9 and Last from 10

With this method you only ever need multiplication tables up to 5 times 5. It is one of many ancient Indian sutras and this one involves a cross subtraction method which, according to old historical traditions, is responsible for the acceptance of the ´ mark as the sign of multiplication. Here is a very simple example of the method. Can you give a good explanation of WHY it works?

Suppose we want to multiply 9 by 7. We subtract each number from 10 and, using these differences (or deficiencies), write:

9-1
7-3
6 3

The product has two parts, left and right.

To get the right part (or units digit) multiply the deficiencies (1×3).

The left hand digit (tens digit) of the answer can be found in four different ways. Why do they all give the same answer?

1. Subtract 10 from the sum of the two given numbers (9+7=16, 16-10=6)
2. Subtract the sum of the two deficiencies (1+3=4) from 10 and you get 6.
3. Cross subtract (9-3=6)
4. Cross subtract (7-1=6)

This gives the answer 63.

Here are some more examples. Try some of your own.

9-1	8-2	9-1	8-2
6-4	7-3	9-1	5-5
5 4	5 6	8 1	4 0
			Note: Here you have to express 5 times 2 as 1 ten and 0 units.

Therapies and Techniques

In keeping with the Life Positive philosophy to open the doors to all schools of wisdom, and to all therapies and techniques that aid well-being, the Expo presented a generous smorgasbord of experiences, systems of knowledge and viewpoints that could not fail to have stimulated, enlightened and inspired. LP reader Chhaya Kothari probably speaks for many participants when she confides, "I came all the way from Rishikesh especially for this event. Rishikesh, of course, hosts many discourses and programmes too, but nowhere is such a varied mix presented. I have thoroughly enjoyed myself here and am truly glad I came."

The most heartwarming aspect of the event and one which has won our eternal gratitude is the number of LP readers who came from all over India to participate. One such was Tony Bonham from Goa who said, "Looking at the event list, I decided I must make it here and I must say, I am truly impressed. Really glad I came." We are really glad you came too, sir! Another was Father Peter from the Father

Agnel Ashram in Pune, who came with a colleague, Father Ivan Almeida. Says the affable Father, "Life Positive Expo-2005 was fantastic.

The three days were like being in a retreat. Most of the exponents of various techniques were good. Swami Veda Bharati was very inspiring." Each day's programme was flagged off at 8 am, with a salutation to the sun in the form of sun yoga by our very own Sun yogi, Umasankarji. Most people who came the first day, continued on all the three days, bringing their friends and relatives, truly inspired by the amazing Sun yogi.

Throughout the three days, participants thronged the workshops, expanding their minds with such concepts as time-line therapy, which demonstrates the vital need for clarity on the past, present and future. Freedom from past 'baggage' through time-line therapy can focus attention on the present, affirmed Dr. Somesh Chadda. Dr. Ketan Shah's ability to heal people of neck aches, backaches, stomach disorders and knee pain in a minute through the application of acupressure, drew a long queue of patients. "This is not a miracle," smiled Dr. Shah, "Just the application of a time-tested proven technique."

The stalls too did brisk business. Tarot reader Sumita Somany had many takers as did AAC-Sukoon, whose stress-relieving armchairs were in great demand. Energy healing through various modalities at Rodsie Realms was also a huge draw. The Swami Rama ashram had a fascinating range of books by the revered sage, including his immensely popular, Living with the Himalayan Masters, eagerly procured by seekers. Other bookstalls such as Wisdom Tree, Pustak Mahal and Happy Thoughts, also drew immense response. Sangeeta Gupta's Money Mantra stall attracted much attention as did the Pranic Healing stall, which saw a steady stream of people being cleansed of their negative auras. Shahnaz Husain's herbal cosmetics also attracted attention and the Nirankari Satsang Mandal did a double whammy, taking both a stall as well as presenting a wonderful workshop with bhajans, poems and discourses, conveying the message of realisation of Nirankar with devotion and fervour.

On a more intellectual plane were our two panel discussions. Both drew packed halls and proved to be stimulating, thought-provoking and inspiring. The sublime insights shared by the panelists of 'What is enlightenment' were enough to send listeners into a mild ecstasy. Vedic scholar Swami Veda Bharati beautifully affirmed that true enlightenment is reflected in humility - in how low one can make oneself.

Islamic scholar Maulana Wahiduddin Khan aptly honed in on anger management as a signal of enlightenment. Prashant Khanna of the Sri Aurobindo Ashram quoted extensively from the Bhagawad Gita to outline the qualities of the individual with stithprajna. Moderator Rajiv Mehrotra, himself a Buddhist practitioner, skilfully wielded the threads of the discussion and brought it to a fitting conclusion.

The other discussion on 'Allopathy and alternatives: Points of convergence' was a very enlightening and inspiring interaction between four allopaths who were firmly in favour of alternative approaches. Sounding more like gurus than sceptical medicos, the quartet agreed fervently on the need for a broader and more holistic approach to healing.

Dr. H. K. Chopra affirmed that alternatives were the real thing and allopathy was the alternative. Dr. Ashwini Chopra, co-owner of Aashlok, divulged that he routinely advised his patients to take up yoga and meditation and no preliminary investigation was complete without inquiring into the patient's emotional state. Dr. Newton and Dr. Tuli also made very relevant points, enthralling the audience with their open-mindedness.

But the highlight of the Expo was indubitably the series of evening programmes. The programme for the first day began with mellifluous and heart-stirring bhajans by Chitra Roy. Starting with the great Shankaracharya's Chidananda Roopam, she went on to sing some of the bhajans popular among the Art of Living followers such as Radhe Shyam.

Then came a tour de force discourse by the inimitable Sri Sri Ravi Shankar, whose fan following girdles the globe. One devotee tailed him all the way from Kenya and was in emotional transports at sight of her guru. He started with a simple and profound observation, "The centre of the atom is positive and negative electrons only exist in the periphery. Similarly, at our core we are positive and only need to remove the superficial negativity." Saying this he gratifyingly eulogised Life Positive's own role in stoking positive values in society and in creating awareness of the great plurality of paths, techniques and healing systems. Such knowledge is essential to guide people on the spiritual path, he said. He also congratulated the magazine for having taken giant steps in growth and becoming the leading purveyor of spiritual values in the country today.

The second day's evening programme started off with a charming Bharata Natyam performance by Marie Elangovan and her students, D. Ayushree Moorthy and S. Aparna. Although the lithe and lissome Ms Elangovan was very impressive, her tiny students, less than nine years old, carried the day with their talent and enthusiasm.

Vedic scholar and international meditation teacher, Swami Veda Bharati, then proceeded to give a rivetting lecture on the qualities of the Indian mind. First deploring the tendency among most of us to embark on this exercise as a preliminary to boosting our ego at the cost of other nations, he reiterated his commitment to global citizenship and emphasised the need for a broad all-inclusive perspective. To introduce the audience to the Indian mind, he requested them to be guided by him into a meditation for it was only through the meditative mind that one could access the nature of the Indian mind. Interspersing his lecture with quotations, anecdotes and incidents that revealed the fathomless depths of his mind, he concluded by saying that one of the most remarkable aspects of the Indian mind was its proclivity for 'also'. The Indian mind declines to be limited by either-or, and insists on saying this and that, he said. The third day's programme was once again entirely different in character. It went off to a rollicking start with joyful and exuberant songs in English from the Ananda Sangha. Aptly called Philosophy in Song, the songs, written by their founder Swami Kriyananda, articulated love of life, love of nature and love of God.

The redoubtable Anandmurti Gurumaa then took the stage and held the audience enthralled with her deep and powerful insights into spirituality and why it is best practised within the context of our day-to-day lives. First emphasising the need for all seekers to free themselves from the outer garb of religion and to immerse themselves in its spiritual core, she went on to enumerate the story of someone who goes to a master and asked him how he could find God. The master looked at him and said, 'Certainly I will help you find Him but first you have to give me the full details, including time and place, of when you lost Him!' At one point she broke into a full-throated rendition of an Urdu couplet that spoke beautifully and movingly about a man who went to a sage simply out of curiosity and found to his astonishment that he was changed for life. Her powerful and mellifluous voice is a phenomenon not easily found and one can well understand why her cassettes are such sell-outs.

And with that we reluctantly came to the end of our soul-satisfying and deeply gratifying programme. To us, the Life Positive team, the Expo was an exhilarating experience, an opportunity to meet and bond with our readers and fellow seekers. It is with deep humility and gratitude that we acknowledge your many generous compliments about Life Positive. We are grateful that you find the magazine of benefit, and that you enjoyed the event. We want you to know that you, the reader, are the sole inspiration for our existence and ongoing activities. In serving you, we hope to scale fresh peaks of excellence.

Vedic Mathematics Today

Having recently read several articles published in India which are critical of Vedic mathematics it seems appropriate to give a response based on the work carried out within the United Kingdom. I have taught Vedic mathematics for almost twenty years at an independent school in London. Some of the students have Indian ancestry but this is not the reason for our approach to mathematics. All our mathematics teachers have become familiar with the system and use it in their lessons. We regularly hold training workshops for the staff and have written our own text books for resource material. Our associate schools in different parts of the world also teach Vedic mathematics. Both teachers and students have found Vedic mathematics of great benefit because it has so many positive qualities. The most obvious of these are found through experience of working with the sutras. Children and adults alike find the system profound, inspiring and delightful. We have also found that the students gain exceptional examination results.

What is Vedic mathematics really about? What is it about the subject that an English school should use it so thoroughly? Is it really Vedic and what was the purpose of Sri Bharati Krishna Tirthaji's book on the subject? In this account I attempt to answer some of these questions.

Vedic mathematics is based on sixteen sutras together with a similar number of sub-sutras. Each sutra provides a principle of mental working applicable to many diverse areas of mathematics. There are several reasons for the title

Vedic but before considering these it is worth mentioning that the word Veda generally has two meanings. The first is the collection of ancient Indian texts relating to both spiritual and secular knowledge. Due to the once strong oral tradition by which these texts were passed on it is not possible to accurately place a date when they were first composed. This meaning of Veda is the most common. The second way that Veda is used is to describe true knowledge in the present which resides within peoples hearts or minds. This is the meaning which we have accepted. This is just as well since Sri Tirthaji bequeathed to us no accurate or definite reference for the sutras. Locating the sutras within ancient texts has so far proven to be beyond some of the most able scholars in this field. It is quite possible that Sri Tirthaji intuited the sutras from his deep understanding of the subject, the Vedas and the nature of the human mind.

Arising from the first of these two meanings of Veda historians of mathematics have tried to relate the work of Sri Tirthaji to the mathematics of well-known ancient texts. If only this first meaning is accepted then the question as to whether Tirthaji's system is or is not Vedic becomes an almost insurmountable task. In my experience it is better to approach Vedic mathematics from the second meaning relating to natural laws working within the human psyche. This is a practical approach and certainly most of the work in the UK has followed this line. We have sympathy for the historians but regard any discovery of reference to the sutras as a bonus rather than a necessity.

The introduction to Vedic Mathematics indicates that during the early part of the 20th century Sri Tirthaji rediscovered or reconstructed Vedic mathematics from stray references within the appendix portions of the Atharvaveda. We do not know whether or not these appendices were published. He evidently spent a large proportion of his life teaching the system but it was only shortly before he passed away that he set down an illustrative volume on the subject.

This was published posthumously in 1965 and is the main source of all the serious study on the subject. His book offers a snapshot of the sutraic system. Some of the sutras are applied to relatively elementary topics in arithmetic and algebra giving rise to fast and easy methods of calculation. Some of these methods are developed quite carefully particularly at the beginning of the book. A few sutras are only mentioned once or twice in rather abstruse and specialised topics. The really surprising aspect is contained within one of his introductions where he describes these few sutras as having jurisdiction over the whole of mathematics.

Years ago when I was first involved with various groups studying Vedic mathematics we all thought this statement outrageous and absurd. How could sixteen sutras apply to the whole of mathematics? Our view was strengthened by the text due to the paucity of explanation of some of the rules. For example, there is a sutra, Vyashti Samashti, which is mentioned only once in the text and even then it is given in relation to a very particular type of biquadratic equation. It is the sort of equation that you are highly unlikely to meet when working in mathematics. Nevertheless he uses this equation as the means to provide us with a brief glimpse at how the sutra can be applied. As it turns out this sutra is fundamental to mathematics particularly in statistics and mechanics. It has countless applications because it describes a common mental process. The sutra means Individuality and totality or Being many, being one or Specific and general. One of the simplest applications occurs when finding the mean or average of a set of numbers. The idea is that an average in some way is a single number representing the whole group. The sutra also carries a much deeper meaning relating to the individual and universal Self. Expressed as above — so below, it is recognised as one of the fundamental philosophical tenets of Western culture. It finds expression within the art and architecture of ancient Egypt, the philosophy of Plato, both the Judaic and Christian teachings and within the Humanism of the Florentine Renaissance. Many of the sutras are like this. They go beyond the purely mathematical sphere and deal with a more fundamental level of human experience.

Going back in time when we first came across Vedic mathematics in London we were impressed by the methods of calculation. They had a different quality to what we were brought up with. Here were methods that were enjoyable, quick, light, easy and refreshing. They related the numbers to their source, unity. They brought the mind out of the wrangles of the past and into the immediacy of the present moment. We found the sutras really brought the subject alive and we still find that all students feel more alive by practising Tirthaji's methods.

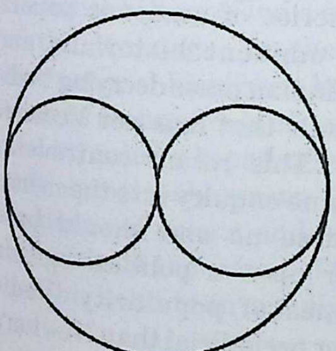
Once our enthusiasm was kindled we studied and practiced the whole of his book. We worked through every sum and read and re-read every word to try and make sense of the system. It became clear that since Sri Tirthaji had only left a specimen we would have to see how the sutras could be applied to other areas of mathematics. What if his statement about the jurisdiction of the sutras was true? Could it be possible that so few rules can take account of all mathematics?

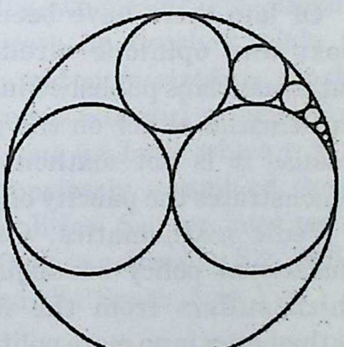
We decided to put his statement to the test and see if there were other applications not mentioned in his book. Initially this was difficult but after a while we discovered a fast method of calculation based on one of the sutras. Over a period of years this work continued and we gradually began to see more and more applications to fast methods of calculation, algebraic manipulations and geometrical theorems. We also discovered the sutras at work within entirely conventional methods, systems and algorithms. For example, although we had found a fast method of subtraction of numbers based on one sutra we also found the correct sutra for the conventional paying and borrowing back method.

This work in turn led to greater understanding of the individual sutras together with a greater confidence in the system. It appeared that the sutras described the mental processes or laws that naturally operate within the mind. The implication is that the human mind has only a limited number of natural processes, channels or principles - each having a myriad of applications.

The next step was to consider topics within mathematics and simply ask, what sutra is working here? For example, what is the sutra working when you bisect an angle or when simplifying an irrational number? The elementary topics are fairly straightforward but what about more sophisticated mathematics? We recently worked through various solutions to the 'shoe-maker's knife' problem in geometry. This is an ancient problem but which has become quite a popular area during the last sixty years or so. The mathematics involved is not easy.

You are given two circles drawn inside a larger circle so that they all touch each other. The area between the circles is the Arbelos or shoe-maker's knife. The problem is to construct further circles within the Arbelos as shown in the diagram. We worked through some of the conventional solutions to this. There was one particularly elegant solution that seemed the quickest and easiest method. It required transformations of a series of circles.





Whilst looking at this solution it dawned on us that the sutra involved was none other than Transpose and adjust, one of the most common sutras in this Vedic system. This is by way of example and illustrates the way in which a simple principle can have an overriding influence over the specific details of the problem.

These anecdotal instances help describe the nature of what we see as Vedic mathematics. On the one hand you have a series of fast, interesting and delightful algorithms and on the other the sutras supply us with a universal approach by which any mathematical process can be perceived as an integral part of a larger whole.

By looking at Tirthaji's system from this point of view we have found that the applications of the sutras are not time dependent. In other words they can be applied and are entirely valid not only to known mathematics from Indian antiquity as well as the mathematics that the Shankarcarya himself had to deal with but also modern topics ranging from the theory of matrices to the theories of chaos and catastrophe. The system Tirthaji described and what has been developed from his work is Vedic by way of function independently of historical background. We have found it of immense practical value. It stands as a bona fide system supported by deep logic. This is why it succeeds in the educational ethos of the United Kingdom and elsewhere.

Of late there have been a series of academic papers, books and opinions aired by eminent historians and mathematicians published in the Indian press decrying Vedic mathematics either on the grounds that it is not Vedic or because it is not mathematics. This recent controversy demonstrates the paucity of genuine enquiry into the merits of Vedic mathematics. Of course no one should base educational policy on dogma or upon a political agenda which suffers from the fortunes of popularity. Vedic mathematics is no more political or prejudicial than Newton's three laws of motion. This can be testified by those outside India who use Vedic mathematics freely and without any cultural association. Nevertheless, it is philosophical since it deals with knowledge and natural law.

Those responsible for developments in education must face practical issues. If, in India, there is a demand for Vedic mathematics in schools then this must be met with the tasks of training teachers, supplying resource materials and altering syllabuses according to a more fundamental and unified system of logic. I hope that every effort can be made to pursue the study of Vedic mathematics in a practical and open-hearted spirit.

Vedic Mathematicians in Ancient India

Uncovering the scope of Ancient Indian Mathematics faces a twofold difficulty. To determine who discovered what we must have an accurate idea of the chronology of Ancient India. This has been made doubly difficult by the faulty dating of Indian Historical events by Sir William Jones, who practically invented the fields of linguistics and philology if for a moment we discount the contributions of Panini (Ashtadhyayi) and Yaska (Nirukta) a couple of millennia before him.

Sir William, who was reputed to be an accomplished linguist, was nevertheless totally ignorant of Sanskrit when he arrived in India and proceeded in short order to decipher the entire history of India from his own meager understanding of the language, In the process he brushed aside the conventional history as known and memorized by Sanskrit pundits for hundreds of years and as recorded in the Puranas and invented a brand new timeline for India which was not only egregiously wrong but hopelessly scrambled up the sequence of events and personalities. See for instance my chronicle on the extent of the damage caused by Sir William and his cohorts in my essay on the South Asia File.

It is not clear whether this error was one caused by inadequate knowledge of language or one due to deliberate falsification of records. It is horrific to think that a scholar of the stature of sir William would resort to skullduggery merely to satisfy his preconceived notions of the antiquity of Indic contributions to the sum of human knowledge. Hence we will assume Napoleon's dictum was at play here and that we should attribute not to malice that which can be explained by sheer incompetence. This mistake has been compounded over the intervening decades by a succession of British historians, who intent on reassuring themselves of their racial superiority, refused to acknowledge the antiquity of India, merely because 'it could not possibly be'. When once they discovered the antiquity of Egypt, Mesopotamia and Babylon, every attempt was made not to disturb the notion that the Tigris Euphrates river valley was the cradle of civilization.

When finally they stumbled upon increasing number of seals culminating in the discovery of Mohenjo Daro and Harappa by Rakhal Das Banerjee and Daya Ram Sahni, they hit upon the ingenious idea that the Vedic civilization and the Indus Valley Civilization or the Saraswathi Sindhu Civilization, a more apt terminology since most of the archaeological sites lie along the banks of the dried up Saraswathi river, were entirely distinct and unrelated to each other. The consequences of such a postulate have been detailed in the South Asia File.

The second difficulty was the Euro centricity (a euphemism for a clearly racist attitude) of European mathematicians, who refused to appreciate the full scope of the Indic contributions and insisted on giving greater credit to Greece and later to Babylonian mathematics rather than recognize Indic and Vedic mathematics on its own merits. If this was indeed a surprise revelation, I fail to see the irony, when a similar Euro centricity was exhibited towards the antiquity of the Vedic people themselves.

An Overview of Indian Mathematics

It is without doubt that mathematics today owes a huge debt to the outstanding contributions made by Indian mathematicians over many hundreds of years. What is quite surprising is that there has been a reluctance to recognize this and one has to conclude that many famous historians of mathematics found what they expected to find, or perhaps even what they hoped to find, rather than to realize what was so clear in front of them.

We shall examine the contributions of Indian mathematics in this article, but before looking at this contribution in more detail we should say clearly that the "huge debt" is the beautiful number system invented by the Indians on which much of mathematical development has rested. Laplace put this with great clarity:-

The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius.

We shall look briefly at the Indian development of the place-value decimal system of numbers later in this article and in somewhat more detail in the separate article Indian numerals. First, however, we go back to the first evidence of mathematics developing in India.

Histories of Indian mathematics used to begin by describing the geometry contained in the Sulvasutras but research into the history of Indian mathematics has shown that the essentials of this geometry were older being contained in the altar constructions described in the Vedic mythology text the Shatapatha Brahmana and the Taittiriya Samhita. Also it has been shown that the study of mathematical astronomy in India goes back to at least the third millennium BC and mathematics and geometry must have existed to support this study in these ancient times.

Evidence from Europe that India is the True Birthplace of Our

Numerals

The views of savants and learned scholars from a non-Indian tradition about Indian mathematics are presented here. Note that most of these are dated prior to the1800's, when India was still untainted with the prefix of being a colonized country

Severus Sebokt of Syria in 662 CE: (the following statement must be understood in the context of the alleged Greek claim that all mathematical knowledge emanated from them

> *"I shall not speak here of the science of the Hindus, who are not even Syrians, and not of their subtle discoveries in astronomy that are more inventive than those of the Greeks and of the Babylonians; not of their eloquent ways of counting nor of their art of calculation, which cannot be described in words - I only want to mention those calculations that are done with nine numerals. If those who believe, because they speak Greek, that they have arrived at the limits of science, would read the Indian texts, they would be convinced, even if a little late in the day, that there are others who know something of value". (Nau, 1910)*

Said al-Andalusi, probably the first historian of Science who in 1068 wrote Kitab Tabaqut al-Umam in Arabic (Book of Categories of Nations).

To their credit, the Indians have made great strides in the study of numbers (3) and of geometry. They have acquired immense information and reached the zenith in their knowledge of the movements of the stars (astronomy) and the secrets of the skies (astrology) as well as other mathematical studies. After all that, they have surpassed all the other peoples in their knowledge of medical science and the strengths of various drugs, the characteristics of compounds and the peculiarities of substances.

Albert Einstein in the 20th century also comments on the importance of Indian arithmetic: "We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made."

Quotes from Liberabaci (Book of the Abacus) by Fibonacci (1170-1250): The nine Indian numerals are ...with these nine and with the sign 0 which in Arabic is sifr, any desired number can be written. (Fibonacci learnt about Indian numerals from his Arab teachers in North Africa) .Fibonacci introduced Indian numerals into Europe in 1202CE.

G Halstead:

> *...The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habituation and a name, a picture, a symbol but helpful power, is the characteristic of the Hindu race from whence it sprang. No single mathematical creation has been more potent for the general on go of intelligence and power. [CS, P 5]*

The Following Quotes are from George Ifrah's Book Universal History of Numbers

The real inventors of this fundamental discovery, which is no less important than such feats as the mastery of fire, the development of agriculture, or the invention of the wheel, writing or the steam engine, were the mathematicians and astronomers of Indian civilisation: scholars who, unlike the

Greeks, were concerned with practical applications and who were motivated by a kind of passion for both numbers and numerical calculations.

There is a great deal of evidence to support this fact, and even the Arabo-Muslim scholars themselves have often voiced their agreement.

The following is a succession of historical accounts in favour of this theory, given in chronological order, beginning with the most recent.

1. P. S. Laplace (1814): "The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius." [Dantzig. p. 26]
2. J. F. Montucla (1798): "The ingenious number-system, which serves as the basis for modern arithmetic, was used by the Arabs long before it reached Europe. It would be a mistake, however, to believe that this invention is Arabic. There is a great deal of evidence, much of it provided by the Arabs themselves that this arithmetic originated in India." [Montucla, I, p. 375J.
3. John Walls (1616-1703) referred to the nine numerals as Indian figures. [Wallis (1695), p. 10]
4. Cataneo (1546) le noue figure de gli Indi, "the nine figures from India". [Smith and Karpinski (1911), p.3.

5. Willichius (1540) talks of Zyphrae! Nice, "Indian figures". [Smith and Karpinski (1911) p. 3]
6. The Crafte of Nombrynge (c. 1350), the oldest known English arithmetical tract: II fforthermore ye most vndirstonde that in this craft ben vsed teen figurys, as here bene writen for esampul 098 ^ 654321... in the quych we vse teen figwys of Inde. Questio II why Zen figurys of Inde? Soiucio. For as I have sayd afore thei werefondefrrst in Inde. [D. E. Smith (1909)
7. Petrus of Dada (1291) wrote a commentary on a work entitled Algorismus by Sacrobosco (John of Halifax, c. 1240), in which he says the following (which contains a mathematical error): Non enim omnis numerus per quascumquefiguras Indorum repraesentatur "Not every number can be represented in Indian figures". [Curtze (1.897), p. 25
8. Around the year 1252, Byzantine monk Maximus Planudes (1260-1310) composed a work entitled Logistike Indike ("Indian Arithmetic") in Greek, or even Psephophoria kata Indos ("The Indian way of counting"), where he explains the following: "There are only nine figures. These are:

   123456789

   [figures given in their Eastern Arabic form]

   A sign known as tziphra can be added to these, which, according to the Indians, means 'nothing'. The nine figures themselves are Indian, and tziphra is written thus: 0". [B. N., Pans. Ancien Fonds grec, Ms 2428, f" 186 r"]
9. Around 1240, Alexandre de Ville-Dieu composed a manual in verse on written calculation (algorism). Its title was Carmen de Algorismo, and it began with the following two lines: Haec algorismus ars praesens dicitur, in qua Talibus Indorumfruimur bis quinquefiguris

"Algorism is the art by which at present we use those Indian figures, which number two times five". [Smith and Karpinski (1911), p. 11]

10. In 1202, Leonard of Pisa (known as Fibonacci), after voyages that took him to the Near East and Northern Africa, and in particular to Bejaia (now in Algeria), wrote a tract on arithmetic entitled Liber Abaci ("a tract about the abacus"), in which he explains the following:

    Cum genitor meus a patria publicus scriba in duana bugee pro pisanis mercatoribus ad earn confluentibus preesset, me in pueritia mea ad se uenire faciens, inspecta utilitate el cornmoditate fiutura, ibi me studio abaci per aliquot dies stare uoluit et doceri. Vbi a mirabii magisterio in arte per nouem figuras Indorum introductus. . . Novem figurae Indorum hae sun!: cum his itaque novemfiguris. et turn hoc signo o. Quod arabice zephirum appellatur, scribitur qui libel numerus:

    "My father was a public scribe of Bejaia, where he worked for his country in Customs, defending the interests of Pisan merchants who made their fortune there. He made me learn how to use the abacus when I was still a child because he saw how I would benefit from this in later life. In this way I learned the art of counting using the nine Indian figures... The nine Indian figures are as follows:

    987654321

    [figures given in contemporary European cursive form].

    "That is why, with these nine numerals, and with this sign 0, called zephirum in Arab, one writes all the numbers one wishes."[Boncompagni (1857), vol.1]

11. C. U50, Rabbi Abraham Ben MeIr Ben Ezra (1092-1167), after a long voyage to the East and a period

spent in Italy, wrote a work in Hebrew entitled: Sefer ha mispar ("Number Book"), where he explains the basic rules of written calculation.

He uses the first nine letters of the Hebrew alphabet to represent the nine units. He represents zero by a little circle and gives it the Hebrew name of galgal ("wheel"), or, more frequently, sfra ("void") from the corresponding Arabic word.

However, all he did was adapt the Indian system to the first nine Hebrew letters (which he naturally had used since his childhood).

In the introduction, he provides some graphic variations of the figures, making it clear that they are of Indian origin, after having explained the place-value system: "That is how the learned men of India were able to represent any number using nine shapes which they fashioned themselves specifically to symbolize the nine units." (Silberberg (1895), p.2: Smith and Ginsburg (1918): Steinschneider (1893)1

12. Around the same time, John of Seville began his Liberalgoarismi de practica arismetrice ("Book of Algoarismi on practical arithmetic") with the following:

Numerus est unitatum cot/echo, quae qua in infinitum progredilur (multitudo enim crescit in infinitum), ideo a peritissimis Indis sub quibusdam regulis et certis lirnitibus infinita numerositas coarcatur, Ut de infinitis dfinita disciplina traderetur etfuga subtilium rerum sub alicuius artis certissima Jege ten eretur:

"A number is a collection of units, and because the collection is infinite (for multiplication can continue indefinitely), the Indians ingeniously enclosed this infinite multiplicity within certain rules and limits so that infinity could be scientifically defined: these strict rules enabled them to pin down this subtle concept.

[B. N., Paris, Ms. lat. 16 202, p 51: Boncompagni (1857), vol. I, p. 261

13. C. 1143, Robert of Chester wrote a work entitled: Algoritmi de numero Indorum ("Algoritmi: Indian figures"), which is simply a translation of an Arabic work about Indian arithmetic. [Karpinski (1915); Wallis (1685). p. 121
14. C. 1140, Bishop Raymond of Toledo gave his patronage to a work written by the converted Jew Juan de Luna and archdeacon Domingo Gondisalvo: the Liber Algorismi de numero Indorum ("Book of Algorismi of Indian figures) which is simply a translation into a Spanish and Latin version of an Arabic tract on Indian arithmetic. [Boncompagni (1857), vol. II
15. C. 1130, Adelard of Bath wrote a work entitled: Algoritmi de numero Indorum ("Algoritmi: of Indian figures"), which is simply a translation of an Arabic tract about Indian calculation. [Boncompagni (1857), vol. II
16. C. 1125, The Benedictine chronicler William of Malmesbury wrote De gestis regum Anglorum, in which he related that the Arabs adopted the Indian figures and transported them to the countries they conquered, particularly Spain. He goes on to explain that the monk Gerbert of Aurillac, who was to become Pope Sylvester II (who died in 1003) and who was immortalized for restoring sciences in Europe, studied in either Seville or Cordoba, where he learned about Indian figures and their uses and later contributed to their circulation in the Christian countries of the West. L Malmesbury (1596), f" 36 r'; Woepcke (1857), p. 35J

17. Written in 976 in the convent of Albelda (near the town of Logroño, in the north of Spain) by a monk named Vigila, the Coda Vigilanus contains the nine numerals in question, but not zero. The scribe clearly indicates in the text that the figures are of Indian origin:

    Item de figuels aritmetice. Scire debemus Indos subtilissimum ingenium habere et ceteras gentes eis in arithmetica et geometrica et ceteris liberalibu.c disciplinis concedere. Et hoc manifèstum at in novem figuris, quibus quibus designant unum quenque gradum cuiu.slibetgradus. Quatrum hec sunt forma:

    9 8 7 6 5 4 3 2 1.

"The same applies to arithmetical figures. It should be noted that the Indians have an extremely subtle intelligence, and when it comes to arithmetic, geometry and other such advanced disciplines, other ideas must make way for theirs. The best proof of this is the nine figures with which they represent each number no matter how high. This is how the figures look:

9 8 7 6 5 4 3 2 1

Al-Khwarismi (783-850 CE) Popularized Indian numerals, mathematics including Algebra in the Islamic world and the Christian West .Algebra was named after his treatise 'Al jabr wa'l Muqabalah" which when translated from Arabic means 'Transposition and Reduction'. Little is known about his life except that he lived at the court of the Abbasid Caliph al Ma'amun , in Baghdad shortly after Charlemagne was made emperor of the west, and that he was one of the most important mathematicians and astronomers who worked at the house of Wisdom (Bayt al Hikma)

'The first mathematics which we shall describe in this article developed in the Indus valley. The earliest known urban Indian culture was first identified in 1921 at Harappa in the Punjab and then, one year later, at Mohenjo-Daro, near the Indus River in the Sindh. Both these sites are now in Pakistan but this is still covered by our term "Indian mathematics" which, in this article, refers to mathematics developed in the Indian subcontinent. The Indus civilisation (or Harappan civilisation as it is sometimes known) was based in these two cities and also in over a hundred small towns and villages. It was a civilisation which began around 2500 BC and survived until 1700 BC or later. The people were literate and used a written script containing around 500 characters which some have claimed to have deciphered but, being far from clear that this is the case, much research remains to be done before a full appreciation of the mathematical achievements of this ancient civilisation can be fully assessed. "

The above statement must be revised based on new archaeological discoveries. More than 400 sites have been found along the banks of the dried up river bed of the ancient river Saraswathi. These sites include the submerged city of Bet Dwaraka, the city ruled by Sri Krishna during the episodes of the Mahabharata and the great Bharata war that is described in detail in that epic. The important point to note is that a prerequisite to do numerical work is a script. So, there must have been a script by the time the Saraswathi Sindhu civilization was flourishing not just centred in the two cities of Mohenjo Daro and Harappa but along dozens of urban towns and cities like Dholavira, Lothal, Dwaraka and others. European historians often wonder what happened to the denizens of the Indus Valley civilization. Ockham's razor suggests the right answer . Nothing catastrophic happened to these people and we the modern Indics are the descendants of this civilization which was spread over a huge area stretching from Haryana in the north to the present day province of Maharashtra to places like Prathishtan (later Pathan) which eventually became the capital of the Satavahana Kingdoms are in fact a successor to the Urban civilizations that existed prior to them. This makes eminent sense because the word Brahmi signifies the goddess Saraswathi (consort of Brahma) and is therefore also considered to be the Guardian deity of Knowledge and the one who is credited with blessing us with the gift of a script. There are a group of Brahmanas in the Konkan area of present day state of Karnataka who call themselves Saraswath Brahmanas and legend has it that they migrated from the banks of the Saraswath river when it eventually dried out. In fact the Gowda Saraswath Brahmanas have done extremely well over the succeeding centuries and have prospered far in excess of their proportion in the population. In fact our family records show that about 15 generations ago my ancestor by the name of Hanuman Bhat migrated to the Andhra country , to escape the turmoil caused by the interminable wars and the tyranny of Aurangazeb , from the area which is considered present day Konkan.

We often think of Egyptians and Babylonians as being the height of civilisation and of mathematical skills around the period of the Indus civilisation, yet V G Childe in New Light on the Most Ancient East (1952) wrote:-

India confronts Egypt and Babylonia by the 3rd millennium with a thoroughly individual and independent civilisation of her own, technically the peer of the rest. And plainly it is deeply rooted in Indian soil. The Indus civilisation represents a very perfect adjustment of human life to a specific environment. And it has endured; it is already specifically Indian and forms the basis of modern Indian culture.

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3

Vedic Mathematics Teaching Methodology

Learning mathematics should be a delightful experience for all children and they should all succeed in it. The Cosmic Computer Course offers a complete system of mental mathematics which can be taught in a holistic way.

The straightforward and beautifully interrelated Vedic methods mean that mathematics can be done mentally and this, together with the many methods of solution which the Vedic system offers, encourages flexibility and innovation.

This in turn leads to the development of creativity and intuition. The Vedic system does not insist on a purely analytic approach as many modern teaching methods do. This makes a big difference to the attitude which children have towards mathematics.

With its direct, easy and integrated approach this mental system (the methods can also be written down) brings out the naturally coherent and unified structure of mathematics.

The Cosmic Computer Course, for example, uses a Unified Field Chart which shows the whole subject at a glance and how the various parts are related and structured.

Vedic Mathematics Sutras*

Sutras	***Subsutras or Corollaries***
1. Ekadhikena Purvena (By One More than the One Before)	Proportionately
2. Nikhilam Navatascharamam Dastah (All from 9 and the Last from 10)	The Remainder Remains Constant
3. Urdhwa-tiryagbhyam (Vertically and Crosswise)	The First by the First and the Last by the Last
4. Paravartya Yojayet (Transpose and Apply)	For 7 the Multiplicand is 143
5. Sunyam Samyasamuchchaye (If the Samuccaya is the Same it is Zero)	By Osculation
6. (Anurupye) Sunyamanyat (If One is in Ratio the Other is Zero)	Lessen by the Deficiency
7. Sankalana-vyavkalanabhyam (By Addition and by Subtraction)	Whatever the Deficiency lessen by that amount and set up the Square of the Deficiency
8. Puranpuranabhyam (By the Completion or Non-Completion)	Last Totalling 10
9. Chalana-Kalanabhyam (Differential Calculus)	Only the Last Terms
10. Yavdunam (By the Deficiency)	The Sum of the Products
11. Vyastisamastih (Specific and General)	By Alternate Elimination and Retention
12. Sesanyankena Charmena (The Remainders by the Last Digit)	By Mere Observation

Contd...

Sutras	*Subsutras or Corollaries*
13. Sopantyadyaymantyam (The Ultimate and Twice the Penultimate)	The Product of the Sum is the Sum of the Products
14. Ekanyunena Purvena (By One Less than the One Before)	On the Flag**
15. Gunitasamuchachayah (The Product of the Sum)	- - -
16. Gunaksamuchchayah (All the Multipliers)	- - -

---

* This list is taken from Vedic Mathematics, which includes a full list of the sixteen Sutras in Sanskrit, but in some cases a translation of the Sanskrit is not given in the text and comes from elsewhere.

** This formula is not in the list given in Vedic Mathematics, but is referred to in the text.

AN EXAMPLE

Urdhwa-tiryagbhyam Sutra

This sutra says -"Vertically and Crosswise". That's all to multiply two numbers!

Till now, you were multiplying like this:

Question : Multiply 432 by 617.

Answer :

```
   432
 X 617
---------
  3024
  432
2592
========
266544
========
```

More the number of digits in the numbers, more lines and time you consume. No more! Using the Sutra "Vertically and Crosswise", you have

Step 1 : (mentally, don't write on notebook) vertically (last digits) :

2 × 7 = 14; write 4 carry 1

Step 2 (mentally) : crosswise (last two digits) :

3 × 7 + 2 × 1 = 23 + carry 1 = 24; write 4 carry 2

Step 3 : vertically and crosswise (three digits) :

4 × 7 + 3 × 1 +2 × 6 = 43 + carry 2 = 45; write 5 carry 4

Step 4 : (move left; first two digits) :

4 × 1 + 3 × 6 = 22 + carry 4 = 26; write 6 carry 2

Step 5 : (move left; first digit of each number) :

4 × 6 = 24 + carry 2 = 26. End.

Write answer : 266544

This is how it appears on notebook :

```
  432
X 617
-------------
266544
-------------
```

No matter how big the numbers are, you will need to write only the final answer. All other steps are easily carried out mentally. If the two numbers have different number of digits, write smaller number below the other and pad it on left side with zeros.

Why Vedic Mathematics?

Many Indian Secondary School students consider Mathematics a very difficult subject. Some students encounter difficulty with basic arithmetical operations. Some students feel it difficult to manipulate symbols and balance equations. In other words, abstract and logical reasoning is their hurdle.

Many such difficulties in learning Mathematics enter into a long list if prepared by an experienced teacher of Mathematics. Volumes have been written on the diagnosis of 'learning difficulties' related to Mathematics and remedial techniques. Learning Mathematics is an unpleasant experience to some students mainly because it involves mental exercise.

Of late, a few teachers and scholars have revived interest in Vedic Mathematics which was developed, as a system derived from Vedic principles, by Swami Bharati Krishna Tirthaji in the early decades of the 20th century.

Dr. Narinder Puri of the Roorke University prepared teaching materials based on Vedic Mathematics during 1986 - 89. A few of his opinions are stated hereunder:

(i) Mathematics, derived from the Veda, provides one line, mental and super- fast methods along with quick cross checking systems.

(ii) Vedic Mathematics converts a tedious subject into a playful and blissful one which students learn with smiles.

(iii) Vedic Mathematics offers a new and entirely different approach to the study of Mathematics based on pattern recognition. It allows for constant expression of a student's creativity, and is found to be easier to learn.

(iv) In this system, for any problem, there is always one general technique applicable to all cases and also a number of special pattern problems. The element of choice and flexibility at each stage keeps the mind lively and alert to develop clarity of thought and intuition, and thereby a holistic development of the human brain automatically takes place.

(v) Vedic Mathematics with its special features has the inbuilt potential to solve the psychological problem of Mathematics - anxiety.

J. T. Glover (London, 1995) says that the experience of teaching Vedic Mathematics' methods to children has shown that a high degree of mathematical ability can be attained from an early stage while the subject is enjoyed for its own merits.

A. P. Nicholas (1984) puts the Vedic Mathematics system as 'one of the most delightful chapters of the 20th century mathematical history'.

Prof. R. C. Gupta (1994) says 'the system has great educational value because the Sutras contain techniques for performing some elementary mathematical operations in simple ways, and results are obtained quickly'.

Prof. J. N. Kapur says 'Vedic Mathematics can be used to remove math-phobia, and can be taught to (school) children as enrichment material along with other high speed methods'.

Dr. Michael Weinless, Chairman of the Department of Mathematics at the M.I.U, Iowa says thus: 'Vedic Mathematics is easier to learn, faster to use and less prone to error than conventional methods. Furthermore, the techniques of Vedic Mathematics not only enable the students to solve specific mathematical problems; they also develop creativity, logical thinking and intuition.'

Keeping the above observations in view, let us enter Vedic Mathematics as given by Sri Bharati Krishna Tirthaji (1884 - 1960), Shankaracharya of Govardhana Math, Puri. Entering into the methods and procedures, one can realize the importance and applicability of the different formulae (Sutras) and methods.

Vedic Mathematical Formulae

What we call Vedic Mathematics is a mathematical elaboration of 'Sixteen Simple Mathematical formulae from the Vedas' as brought out by Sri Bharati Krishna Tirthaji. In the text authored by the Swamiji, nowhere has the list of the Mathematical formulae (Sutras) been given. But the Editor of the text has compiled the list of the formulae from stray references in the text. The list so compiled contains Sixteen Sutras and Thirteen Sub - Sutras as stated hereunder.

Sixteen Sutras

1. Ekddhikena Purena (also a corollary)
2. Nikhilam Navatascaramam Dasatah
3. Urdhva-tiryagbhyam
4. Paravarya Yojayet
5. Sunyam Samyasamuccaye
6. (Anurupye) Sunyamanyat
7. Sankalana-vyavakalana-bhyam (also a corollary)
8. Puranapuranabhyam
9. Calana-Kalanabhyam
10. Yavadunam
11. Vyastisamastih
12. Sesanyankena Caramena
13. Sopantyadvayamantyam
14. Ekanyunena Purvena
15. Gunitasamuccayah
16. Gunakasamuccayah.

Thirteen Sub-sutras

1. Anurupyena
2. sisyate Sesasamjnah
3. Adyamadyenantya-mantyena

4. Kevalaih Saptakam Gunyat
5. Vestanam
6. Yavadunam Tavadunam
7. Yavadunam Tavadunikrtya Varganca Yojayet
8. Antyayordasakepi
9. Antyayoreva
10. Samuccayagunitah
11. Lopanasthapanabhyam
12. Vilokanam
13. Gunitasamuccayah Samuccayagunitah.

In the text, the words Sutra, aphorism, formula are used synonymously. So are also the words Upa-sutra, Sub-sutra, Sub-formula, corollary used.

Now we shall have the literal meaning, contextual meaning, process, different methods of application along with examples for the Sutras. Explanation, methods, further short-cuts, algebraic proof, etc. follow. What follows relates to a single formula or a group of formulae related to the methods of Vedic Mathematics.

Ekadhikena Purvena

The Sutra (formula) Ekadhikena Purvena means: "By one more than the previous one".

(i) Squares of numbers ending in 5 :

Now we relate the sutra to the 'squaring of numbers ending in 5'. Consider the example 25^2.

Here the number is 25. We have to find out the square of the number. For the number 25, the last digit is 5 and the 'previous' digit is 2. Hence, 'one more than the previous one', that is, 2+1=3. The Sutra, in this context, gives the procedure 'to multiply the previous digit 2 by one more than itself, that is, by 3'. It becomes the L.H.S (left hand side) of the result, that is, 2 × 3 = 6. The R.H.S (right hand side) of the result is 52, that is, 25.

Thus $25^2 = 2 \times 3 / 25 = 625$.

In the same way,

$35^2 = 3 \times (3+1) /25 = 3 \times 4/25 = 1225$;

$652 = 6 \times 7 / 25 = 4225$;

$1052 = 10 \times 11/25 = 11025$;

$1352 = 13 \times 14/25 = 18225$;

Apply the formula to find the squares of the numbers 15, 45, 85, 125, 175 and verify the answers.

ALGEBRAIC PROOF:

(a) Consider $(ax + b)^2 \equiv a^2 . x^2 + 2abx + b^2$.

This identity for x = 10 and b = 5 becomes

$$\begin{aligned}(10a + 5)\ 2 &= a^2 . 10^2 + 2. 10a . 5 + 5^2\\ &= a^2 . 10^2 + a. 10^2 + 5^2\\ &= (a^2 + a) . 10^2 + 5^2\\ &= a\ (a + 1) . 10^2 + 25.\end{aligned}$$

Clearly 10a + 5 represents two-digit numbers 15, 25, 35, -------, 95 for the values a = 1, 2, 3, -------, 9 respectively. In such a case the number $(10a + 5)^2$ is of the form whose L.H.S is a (a + 1) and R.H.S is 25, that is, a (a + 1) / 25.

Thus any such two digit number gives the result in the same fashion.

EXAMPLE:

$45 = (40 + 5)^2$, It is of the form $(ax+b)^2$ for a = 4, x=10

and b = 5. giving the answer a (a+1) / 25

that is, 4 (4+1) / 25 + 4 × 5 / 25 = 2025.

(b) Any three digit number is of the form $ax^2 + bx + c$ for $x = 10$, $a \neq 0$, $a, b, c \in W$.

Now $(ax^2+bx+ c)^2 = a^2 x^4 + b^2x^2 + c^2 + 2abx^3$

$+ 2bcx + 2cax^2$

$= a^2x^4 + 2ab.x^3 + (b^2 + 2ca)x^2$

$+2bc \ . \ x + c^2$.

This identity for $x = 10$, $c = 5$ becomes $(a.10^2 + b .10 + 5)^2$

$= a^2.10^4 + 2.a.b.10^3 + (b^2 + 2.5.a)10^2 + 2.b.5.10 + 5^2$

$= a^2.10^4 + 2.a.b.10^3 + (b^2 + 10 \ a)10^2 + b.10^2 + 5^2$

$= a^2.10^4 + 2ab.10^3 + b^2.10^2 + a.10^3 + b.10^2 + 5^2$

$= a^2.10^4 + (2ab + a).10^3 + (b^2 + b)10^2 + 5^2$

$= [a^2.10^2 + 2ab.10 + a.10 + b^2 + b] \ 10^2 + 5^2$

$= (10a + b) \ (10a + b + 1).10^2 + 25$

$= P \ (P+1) \ 10^2 + 25$, where $P = 10a+b$.

Hence any three digit number whose last digit is 5 gives the same result as in (a) for $P=10a + b$, the 'previous' of 5.

EXAMPLE:

$1652 = (1.10^2 + 6.10 + 5)^2$.

It is of the form $(ax^2 + bx + c)^2$ for $a = 1$, $b = 6$, $c = 5$ and $x = 10$. It gives the answer $P(P+1) / 25$, where $P = 10a + b = 10 \times 1 + 6 = 16$, the 'previous'. The answer is $16 \ (16 + 1) / 25 = 16 \times 17 / 25 = 27225$.

Apply Ekadhikena purvena to find the squares of the numbers 95, 225, 375, 635, 745, 915, 1105, 2545.

(ii) Vulgar fractions whose denominators are numbers ending in NINE :

We now take examples of 1 / a9, where a = 1, 2, -----, 9. In the conversion of such vulgar fractions into recurring decimals, Ekadhika process can be effectively used both in division and multiplication.

(a) *Division Method* : Value of 1 / 19.

The numbers of decimal places before repetition is the difference of numerator and denominator, i.e., 19 -1=18 places.

For the denominator 19, the purva (previous) is 1.

Hence Ekadhikena purva (one more than the previous) is 1 + 1 = 2.

The sutra is applied in a different context. Now the method of division is as follows:

Step. 1 : Divide numerator 1 by 20.

i.e., 1 / 20 = 0.1 / 2 = .10 (0 times, 1 remainder)

Step. 2 : Divide 10 by 2

i.e., 0.005 (5 times, 0 remainder)

Step. 3 : Divide 5 by 2

i.e., 0.0512 (2 times, 1 remainder)

Step. 4 : Divide 12 i.e., 12 by 2

i.e., 0.0526 (6 times, No remainder)

144581

Step. 5 : Divide 6 by 2

i.e., 0.05263 (3 times, No remainder)

Step. 6 : Divide 3 by 2

i.e., 0.0526311(1 time, 1 remainder)

Step. 7 : Divide 11 i.e., 11 by 2

i.e., 0.05263115 (5 times, 1 remainder)

Step. 8 : Divide 15 i.e., 15 by 2

i.e., 0.052631517 (7 times, 1 remainder)

Step. 9 : Divide 17 i.e., 17 by 2

i.e., 0.05263157 18 (8 times, 1 remainder)

Step. 10 : Divide 18 i.e., 18 by 2

i.e., 0.0526315789 (9 times, No remainder)

Step. 11 : Divide 9 by 2

i.e., 0.0526315789 14 (4 times, 1 remainder)

Step. 12 : Divide 14 i.e., 14 by 2

i.e., 0.052631578947 (7 times, No remainder)

Step. 13 : Divide 7 by 2

i.e., 0.05263157894713 (3 times, 1 remainder)

Step. 14 : Divide 13 i.e., 13 by 2

i.e., 0.052631578947316 (6 times, 1 remainder)

Step. 15 : Divide 16 i.e., 16 by 2

i.e., 0.052631578947368 (8 times, No remainder)

Step. 16 : Divide 8 by 2

i.e., 0.0526315789473684 (4 times, No remainder)

Step. 17 : Divide 4 by 2

i.e., 0.05263157894736842 (2 times, No remainder)

Step. 18 : Divide 2 by 2

i.e., 0.052631578947368421 (1 time, No remainder)

Now from step 19, i.e., dividing 1 by 2, Step 2 to Step. 18 repeats thus giving

0 ________________ .

1/19 = 0.052631578947368421 or 0.052631578947368421

Note that we have completed the process of division only by using '2'. Nowhere the division by 19 occurs.

(b) *Multiplication Method:* Value of 1 / 19

First we recognize the last digit of the denominator of the type 1 / a9. Here the last digit is 9.

For a fraction of the form in whose denominator 9 is the last digit, we take the case of 1 / 19 as follows:

For 1 / 19, 'previous' of 19 is 1. And one more than of it is 1 + 1 = 2.

Therefore 2 is the multiplier for the conversion. We write the last digit in the numerator as 1 and follow the steps leftwards.

Step. 1 : 1

Step. 2 : 21(multiply 1 by 2, put to left)

Step. 3 : 421(multiply 2 by 2, put to left)

Step. 4 : 8421(multiply 4 by 2, put to left)

Step. 5 : 168421 (multiply 8 by 2 =16,
1 carried over, 6 put to left)

Step. 6 : 1368421 (6 × 2 =12,+1 [carry over]
= 13, 1 carried over, 3 put to left)

Step. 7 : 7368421 (3 × 2, = 6 +1 [Carryover]
= 7, put to left)

Step. 8 : 147368421 (as in the same process)

Step. 9 : 947368421 (Do - continue to step 18)

Step. 10 : 18947368421

Step. 11 : 178947368421

Step. 12 : 1578947368421

Step. 13 : 11578947368421

Step. 14 : 31578947368421

Step. 15 : 631578947368421

Step. 16 : 12631578947368421

Step. 17 : 52631578947368421

Step. 18 : 1052631578947368421

Now from step 18 onwards the same numbers and order towards left continue.

Thus 1 / 19 = 0.052631578947368421

It is interesting to note that we have

(i) not at all used division process

(ii) instead of dividing 1 by 19 continuously, just multiplied 1 by 2 and continued to multiply the resultant successively by 2.

Observations :

(a) For any fraction of the form 1 / a9 i.e., in whose denominator 9 is the digit in the units place and a is the set of remaining digits, the value of the fraction is in recurring decimal form and the repeating block's right most digit is 1.

(b) Whatever may be a9, and the numerator, it is enough to follow the said process with (a+1) either in division or in multiplication.

(c) Starting from right most digit and counting from the right, we see (in the given example 1 / 19)

Sum of 1st digit + 10th digit = 1 + 8 = 9

Sum of 2nd digit + 11th digit = 2 + 7 = 9

- -

Sum of 9th digit + 18th digit = 9 + 0 = 9

From the above observations, we conclude that if we find first 9 digits, further digits can be derived as complements of 9.

(i) Thus at the step 8 in division process we have 0.052631517 and next step. 9 gives 0.052631578

Now the complements of the numbers

0, 5, 2, 6, 3, 1, 5, 7, 8 from 9

9, 4, 7, 3, 6, 8, 4, 2, 1 follow the right order

i.e., 0.052631578947368421

Now taking the multiplication process we have

Step. 8 : 147368421

Step. 9 : 947368421

Now the complements of 1, 2, 4, 8, 6, 3, 7, 4, 9 from 9 i.e., 8, 7, 5, 1, 3, 6, 2, 5, 0 precede in successive steps, giving the answer.

0.052631578947368421.

(d) When we get (Denominator - Numerator) as the product in the multiplicative process, half the work is done. We stop the multiplication there and mechanically write the remaining half of the answer by merely taking down complements from 9.

(e) Either division or multiplication process of giving the answer can be put in a single line form.

ALGEBRAIC PROOF :

Any vulgar fraction of the form 1 / a9 can be written as

1 / a9 = 1 / ((a + 1) x - 1) where x = 10

$$=\frac{1}{(a+1)+\left[\frac{1-1}{(a+1)x}\right]}$$

$$=\frac{1}{(a+1)x}\left[\frac{1-1}{(a+1)x}\right]^{-1}$$

$$=\frac{1}{(a+1)x}\left[1+\frac{1}{(a+1)}x+\frac{1}{(a+1)}x^{2}+\dots\right]$$

$$=\frac{1}{(a+1)}x+\frac{1}{(a+1)^{2}}x^{2}+\frac{1}{(a+1)^{3}}x^{3}+\dots\dots\text{ ad inf inf initum}$$

$$=10^{-1}\left(\frac{1}{(a+1)}\right)+10^{-2}\left(\frac{1}{(a+1)^{2}}\right)+10^{-3}\left(\frac{1}{(a+1)^{3}}\right)+\dots\text{ ad inf initum}$$

This series explains the process of ekadhik.

Now consider the problem of 1 / 19. From above we get

$1/19 = 10^{-1} (1/(1+1)) + 10^{-2} (1/(1+1)2) + 10^{-3} (1/(1+1)3) +$-

(since a=1)

$= 10^{-1} (1/2) + 10^{-2} (1/2)^2 + 10^{-3} (1/3)^3 +$ ----------

$= 10^{-1} (0.5) + 10^{-2} (0.25) + 10^{-3} (0.125)+$ ----------

= 0.05 + 0.0025 + 0.000125 + 0.00000625+ - - - - - - -

= 0.052631 - - - - - - -

EXAMPLE 1 :

1. Find 1 / 49 by ekadhikena process.

Now 'previous' is 4.

'One more than the previous' is 4 + 1 = 5.

Now by division right ward from the left by '5'.

1 / 49 = .10 - - - - - - - - - - - - -(divide 1 by 50)

= .02 - - - - - - - - - (divide 2 by 5, 0 times, 2 remainder)

= .0220 - - - - - - --(divide 20 by 5, 4 times)

= .0204 - - - - - - -(divide 4 by 5, 0 times, 4 remainder)

= .020440 -- - -- - (divide 40 by 5, 8 times)

= .020408 - - - - - (divide 8 by 5, 1 time, 3 remainder)

= .02040831 - - - -(divide 31 by 5, 6 times, 1 remainder)

= .02040811 6 - - - - - - - - continue

= .020408161332261530611222244448 - -- - - - -

On completing 21 digits, we get 48

i.e.,,Denominator - Numerator = 49 - 1 = 48 stands.

i.e, half of the process stops here. The remaining half can be obtained as complements from 9.

Thus 1 / 49 = 0.020408163265306122448
979591836734693877551

Now finding 1 / 49 by process of multiplication left ward from right by 5, we get

1/49 = --1

= --51

= --2551

= --------------------------------------27551

= ---- 483947294594118333617233446943383727551

i.e.,Denominator - Numerator = 49 - 1

= 48 is obtained as 5X9+3

(Carry over) = 45 + 3 = 48.

Hence half of the process is over. The remaining half is automatically obtained as complements of 9.

Thus1 / 49 = ---------------979591836734693877551

= 0.020408163265306122448

979591836734693877551

EXAMPLE 2:

Find 1 / 39 by Ekadhika process.

Now by multiplication method, Ekadhikena purva is 3 + 1 = 4

1/39 = ------------------------------------1

= -----------------------------------41

= -------------------------------1641

= -----------------------------25641

= ----------------------------225641

= --------------------------1025641

Here the repeating block happens to be block of 6 digits. Now the rule predicting the completion of half of the computation does not hold. The complete block has to be computed by ekadhika process. Now continue and obtain the result. Find reasons for the non-applicability of the said 'rule'.

Nikhilam Navatas 'Caramam Dasatah

The formula simply means : "all from 9 and the last from 10".

The formula can be very effectively applied in multiplication of numbers, which are nearer to bases like 10, 100, 1000 i.e., to the powers of 10 . The procedure of multiplication using the Nikhilam involves minimum number of steps, space, time saving and only mental calculation. The numbers taken can be either less or more than the base considered.

The difference between the number and the base is termed as deviation. Deviation may be positive or negative. Positive deviation is written without the positive sign and the negative deviation, is written using Rekhank (a bar on the number). Now observe the following table.

Number	Base	Number - Base	Deviation
14	10	14 - 10	4
8	10	8 - 10	–2 or $\overline{2}$
97	100	97 - 100	–03 or $\overline{03}$
112	100	112 - 100	12
993	1000	993 - 1000	–007 or $\overline{007}$
1011	1000	1011 - 1000	011

Some rules of the method (near to the base) in Multiplication.

(a) Since deviation is obtained by Nikhilam sutra we call the method as Nikhilam multiplication.

E.g. : 94. Now deviation can be obtained by 'all from 9 and the last from 10' sutra i.e., the last digit 4 is from 10 and remaining digit 9 from 9 gives 06.

(b) The two numbers under consideration are written one below the other. The deviations are written on the right hand side.

E.g. : Multiply 7 by 8.

Now the base is 10. Since it is near

to both the numbers, 7

we write the numbers one below the other. 8

Take the deviations of both the numbers from

the base and represent $7\ \bar{3}$

Rekhank or the minus sign before the deviations $8\ \bar{2}$

------

or 7 -3

8 -2

----

or remainders 3 and 2 implies that the numbers to be multiplied are both less than 10.

(c) The product or answer will have two parts, one on the left side and the other on the right. A vertical or a slant line i.e., a slash may be drawn for the demarcation of the two parts i.e.,

$$\begin{array}{cc} 7 & \bar{3} \\ 8 & \bar{2} \\ \hline & / \\ \hline \end{array} \quad \text{(or)} \quad \begin{array}{cc} 7 & -3 \\ 8 & -2 \\ \hline & / \\ \hline \end{array}$$

(d) The R.H.S. of the answer is the product of the deviations of the numbers. It shall contain the number of digits equal to number of zeroes in the base.

i.e.,
$$\begin{array}{ll} 7 & \bar{3} \\ 8 & \bar{2} \\ \hline / & (3 \times 2) = 6 \\ \hline \end{array}$$

Since base is 10, 6 can be taken as it is.

(e) L.H.S of the answer is the sum of one number with the deviation of the other. It can be arrived at in any one of the four ways.

(i) Cross-subtract deviation 2 on the second row from the original number 7 in the first row i.e., 7-2 = 5.

(ii) Cross-subtract deviation 3 on the first row from the original number 8 in the second row (converse way of (i)) i.e., 8 - 3 = 5

(iii) Subtract the base 10 from the sum of the given numbers. i.e., (7 + 8) - 10 = 5

(iv) Subtract the sum of the two deviations from the base.

i.e., 10 - (3 + 2) = 5

Hence 5 is left hand side of the answer.

Thus
$$\begin{array}{ll} 7 & \bar{3} \\ 8 & \bar{2} \\ \hline 5 & / \\ \hline \end{array}$$

Now (d) and (e) together give the solution

$$\begin{array}{lcr} 7 & & \bar{3} \\ 8 & & \bar{2} \\ \hline 5 & / & 6 \\ \hline \end{array} \text{ i.e., } \begin{array}{r} 7 \\ \times\ 8 \\ \hline 56 \\ \hline \end{array}$$

(f) If R.H.S. contains less number of digits than the number of zeros in the base, the remaining digits are filled up by giving zero or zeroes on the left side of the R.H.S. If the number of digits are more than the number of zeroes in the base, the excess digit or digits are to be added to L.H.S of the answer.

The general form of the multiplication under Nikhilam can be shown as follows : Let N_1 and N_2 be two numbers near to a given base in powers of 10, and D_1 and D_2 are their respective deviations from the base. Then $N_1 \times N_2$ can be represented as

$$\begin{array}{lcl} N_1 & & D_1 \\ N_2 & & D_2 \\ \hline (N_1 + D_2) \text{ or } & / & D_1 \times D_2 \\ (N_2 + D_1) & & \\ \hline \end{array}$$

Case (i)

Both the numbers are lower than the base. We have already considered the example 7 × 8 , with base 10. Now let us solve some more examples by taking bases 100 and 1000 respectively.

EXAMPLE 1

Find 97 × 94. Here base is 100. Now following the rules, the working is as follows:

$$\begin{array}{lcl} 97 & & \overline{03} \\ 94 & & \overline{06} \\ \hline (97 - 06) \text{ or } & / & 3 \times 6 \\ (94 - 03) & & \\ \hline \end{array}$$

EXAMPLE 2

98 × 97 Base is 100.

$$\begin{array}{lcl} 98 & & \overline{02} \\ 97 & & \overline{03} \\ \hline (98 - 03) \text{ or } & / & 2 \times 3 \\ (97 - 02) & & \\ \hline \end{array}$$

EXAMPLE 3

75 × 95. Base is 100.

$$\begin{array}{ll}
75 & \overline{25} \\
95 & \overline{05} \\
\hline
(75-05) \text{ or} & \\
 & / \ 25 \times 5 = 70/125 \text{ (observe rule} - \text{f)} \\
(95-25) & \\
\hline
 & = (70+1)/25 = 7125
\end{array}$$

EXAMPLE 4

986 × 989. Base is 1000.

$$\begin{array}{ll}
986 & \overline{014} \\
989 & \overline{011} \\
\hline
(986-14) \text{ or} & \\
 & / \ 14 \times 11 \ = 975/154 = 975154 \\
(989-14) & \\
\hline
\end{array}$$

EXAMPLE 5

994 × 988. Base is 1000.

$$\begin{array}{ll}
994 & \overline{006} \\
988 & \overline{012} \\
\hline
(994-12) \text{ or} & \\
 & / \ 6 \times 12 \ = 982/072 \text{ (rule} - \text{f)} \\
(988-06) & \\
\hline
 & = 982072
\end{array}$$

EXAMPLE 6

750 × 995.

$$\begin{array}{ll}
750 & \overline{250} \\
995 & \overline{005} \\
\hline
(750-005) \text{ or} & \\
 & / \ 250 \times 6 \ = 745/1250 \text{(rule} - \text{f)} \\
(995-250) & \\
\hline
 & = 746250
\end{array}$$

Case (ii)

Both the numbers are higher than the base.

The method and rules follow as they are. The only difference is the positive deviation. Instead of cross - subtract, we follow cross - add.

EXAMPLE 7

13 × 12. Base is 10

$$\begin{array}{lll} 13 & 03 & \\ 12 & 02 & \\ \hline (13+02) \text{ or } & & \\ (12+03) & /\ \ 3\times 2 & = 15/6 = 156 \\ \hline \end{array}$$

EXAMPLE 8

18 × 14. Base is 10.

$$\begin{array}{lll} 18 & 08 & \\ 14 & 04 & \\ \hline (18+06) \text{ or } & & \\ (14+03) & /\ \ 8\times 4 & = 22/32 (\text{rule} - \text{f}, 3 \text{ carry over}) \\ \hline & & = 252 \end{array}$$

EXAMPLE 9

104 × 102. Base is 100.

$$\begin{array}{lll} 104 & 04 & \\ 102 & 02 & \\ \hline 106 & /\ \ 4\times 2 & = 10608\ (\text{rule} - \text{f}) \\ \hline \end{array}$$

EXAMPLE 10

1275 × 1004. Base is 1000.

$$\begin{array}{lll} 1275 & 275 & \\ 1004 & 004 & \\ \hline 1279 & /\ \ 275\times 4 & = 1279/1100 (\text{rule} - \text{f}) \\ \hline & & = 128100 \end{array}$$

Case (iii)

One number is more and the other is less than the base.

In this situation one deviation is positive and the other is negative. So the product of deviations becomes negative. So the right hand side of the answer obtained will therefore have to be subtracted. To have a clear representation and understanding a vinculum is used. It proceeds into normalization.

EXAMPLE 11

13 × 7. Base is 10

$$\begin{array}{lcl} \quad 13 & & 03 \\ \quad 7 & & \overline{03} \\ \hline (13-03) \text{ or} & / & 3\times\overline{3} \;=10/\overline{9}=100.9=91 \\ (7+03) & & \\ \hline \end{array}$$

Note : Conversion of common number into vinculum number and vice versa.

E.g. :

$$9 = 10 - 1 = 11$$

$$98 = 100 - 2 = 10\overline{2}$$

$$196 = 200 - 4 = 20\overline{4}$$

$$3\overline{2} = 30 - 2 = 28$$

$$14\overline{5} = 140 - 5 = 135$$

$$33\overline{2} = 300 - 22 = 278. \text{ etc.}$$

The procedure can be explained in detail using Nikhilam Navatascaram Dasatah, Ekadhikena purvena, Ekanyunena purvena in the foregoing pages of this book.]

EXAMPLE 12

108 × 94. Base is 100.

$$\begin{array}{lcl} \quad 108 & & 08 \\ \quad 94 & & \overline{06} \\ \hline (108-06) \text{ or} & / & 8\times\overline{6} \;=12/4\overline{8}=10152 \\ (94+08) & & \\ \hline \end{array}$$

(since the complement of 48 is 52 – base 10)

EXAMPLE 13

998×1025. Base is 1000.

$$\begin{array}{lcl} 998 & & \overline{002} \\ 1025 & & \overline{025} \\ \hline \begin{array}{l}(998-25) \text{ or} \\ (1025+2)\end{array} & / & \bar{2}\times 25 \\ \hline \end{array} = 1023/\overline{050} = 1022950$$

(since the complement of 50 is 950 for the base 1000)

ALGEBRAIC PROOF

Case (i)

Let the two numbers N1 and N2 be less than the selected base say x.

$N_1 = (x\text{-}a)$, $N_2 = (x\text{-}b)$. Here a and b are the corresponding deviations of the numbers N_1 and N_2 from the base x. Observe that x is a multiple of 10.

Now $N_1 \times N_2 = (x-a)\ (x-b) = x.x - x.b - a.x + ab$

$= x\ (x - a - b) + ab$. [rule - e(iv), d]

$= x\ [(x - a) - b] + ab = x\ (N_1\text{-}b) + ab$[rule-e(i),d]

or$= x\ [(x - b) - a] = x\ (N2 - a) + ab$. [rule -e (ii),d]

$x\ (x - a - b) + ab$ can also be written as

$x[(x - a) + (x - b) - x] + ab = x[N_1+N_2 - x] + ab$ [rule - e(iii),d].

A difficult can be faced, if the vertical multiplication of the deficit digits or deviations i.e., a.b yields a product consisting of more than the required digits. Then rule-f will enable us to surmount the difficulty.

Case (ii)

When both the numbers exceed the selected base, we have $N_1 = x + a$, $N_2 = x + b$, x being the base. Now the identity $(x+a)\ (x+b) = x(x+a+b) + a.b$ holds good, of course with relevant details mentioned in case (i).

Case (iii)

When one number is less and another is more than the base, we can use (x-a)(x+b) = x(x-a+ b)-ab and the procedure is evident from the examples given.

Find the following products by Nikhilam formula.

1) 7 × 4

2) 93 × 85

3) 875 × 994

4) 1234 × 1002

5) 1003 × 997

6) 11112 × 9998

7) 1234 × 1002

8) 118 × 105

Nikhilam in Division

Consider some two digit numbers (dividends) and same divisor 9. Observe the following example.

(i) 13 ÷ 9 The quotient (Q) is 1, Remainder (R) is 4.

since 9) 13 (1
9
____
4

(ii) 34 ÷ 9, Q is 3, R is 7.

(iii) 60 ÷ 9, Q is 6, R is 6.

(iv) 80 ÷ 9, Q is 8, R is 8.

Now we have another type of representation for the above examples as given hereunder:

(i) Split each dividend into a left hand part for the Quotient and right - hand part for the remainder by a slant line or slash.

E.g..13 as 1 / 3, 34 as 3 / 4 , 80 as 8 / 0.

(ii) Leave some space below such representation, draw a horizontal line.

E.g. 1 / 3 3 / 4 8 / 0

____ , ____ , ____

(iii) Put the first digit of the dividend as it is under the horizontal line. Put the same digit under the right hand part for the remainder, add the two and place the sum i.e., sum of the digits of the numbers as the remainder.

E.g.

$$\begin{array}{r} 1\ /\ 3 \\ 1, \\ \hline 1\ /\ 4 \end{array} \qquad \begin{array}{r} 3\ /\ 4 \\ 3, \\ \hline 3\ /\ 7 \end{array} \qquad \begin{array}{r} 8\ /\ 0 \\ 8 \\ \hline 8\ /\ 8 \end{array}$$

Now the problem is over. i.e.,

$13 \div 9$ gives $Q = 1$, $R = 4$

$34 \div 9$ gives $Q = 3$, $R = 7$

$80 \div 9$ gives $Q = 8$, $R = 8$

Proceeding for some more of the two digit number division by 9, we get

(a) $21 \div 9$ as

$$\begin{array}{rr} 9) & 2\ /\ 1 \\ & 2 \\ \hline & 2\ /\ 3 \end{array} \quad \text{i.e. } Q = 2, R = 3$$

(b) $43 \div 9$ as

$$\begin{array}{rr} 9) & 4\ /\ 3 \\ & 4 \\ \hline & 4\ /\ 7 \end{array} \quad \text{i.e. } Q = 4, R = 7$$

The examples given so far convey that in the division of two digit numbers by 9, we can mechanically take the first digit down for the quotient - column and that, by adding the quotient to the second digit, we can get the remainder.

Now in the case of 3 digit numbers, let us proceed as follows.

$$\text{(i)}\quad \begin{array}{rrl} 9) & 104 & (11 \\ & 99 & \\ \cline{2-2} & 5 & \end{array} \qquad \text{as} \qquad \begin{array}{rrcl} 9) & 10 & / & 4 \\ & 1 & / & 1 \\ \cline{2-4} & 11 & / & 5 \end{array}$$

$$\text{(ii)}\quad \begin{array}{rrl} 9) & 212 & (23 \\ & 207 & \\ \cline{2-2} & 5 & \end{array} \qquad \text{as} \qquad \begin{array}{rrcl} 9) & 21 & / & 2 \\ & 2 & / & 3 \\ \cline{2-4} & 23 & / & 5 \end{array}$$

$$\text{(iii)}\quad \begin{array}{rrl} 9) & 401 & (44 \\ & 396 & \\ \cline{2-2} & 5 & \end{array} \qquad \text{as} \qquad \begin{array}{rrcl} 9) & 40 & / & 1 \\ & 4 & / & 4 \\ \cline{2-4} & 44 & / & 5 \end{array}$$

Note that the remainder is the sum of the digits of the dividend. The first digit of the dividend from left is added mechanically to the second digit of the dividend to obtain the second digit of the quotient. This digit added to the third digit sets the remainder. The first digit of the dividend remains as the first digit of the quotient.

Consider 511 ÷ 9

Add the first digit 5 to second digit 1 getting $5 + 1 = 6$.

Hence Quotient is 56. Now second digit of 56 i.e., 6 is added to third digit 1 of dividend to get the remainder i.e., $1 + 6 = 7$

Thus

$$\begin{array}{rrcl} 9) & 51 & / & 1 \\ & 5 & / & 6 \\ \cline{2-4} & 56 & / & 7 \end{array}$$

Q is 56, R is 7.

Extending the same principle even to bigger numbers of still more digits, we can get the results.

E.g : 1204 ÷ 9

(i) Add first digit 1 to the second digit 2. $1 + 2 = 3$

(ii) Add the second digit of quotient 13. i.e., 3 to third digit '0' and obtain the Quotient. 3 + 0 = 3, 133

(iii) Add the third digit of Quotient 133 i.e., 3 to last digit '4' of the dividend and write the final Quotient and Remainder. R = 3 + 4 = 7, Q = 133

In symbolic form

$$\begin{array}{rrcl} 9) & 120 & / & 4 \\ & 13 & / & 3 \\ \cline{2-4} & 133 & / & 7 \end{array}$$

Another example.

$$\begin{array}{lrcll} \text{gives } 9) & 13210 & / & 1 & \quad 132101 \div 9 \\ & 1467 & / & 7 & \quad Q = 14677, R = 8 \\ \cline{2-4} & 14677 & / & 8 & \end{array}$$

In all the cases mentioned above, the remainder is less than the divisor. What about the case when the remainder is equal or greater than the divisor?

$$\begin{array}{lrcll} \textbf{E.g.}\ 9) & 3 & / & 6 & \\ & & / & 3 & \\ \cline{2-4} & 3 & / & 9 & \text{(equal)} \end{array} \qquad \text{or} \qquad \begin{array}{rrcll} 9) & 24 & / & 6 & \\ & 2 & / & 6 & \\ \cline{2-4} & 26 & / & 12 & \text{(greater).} \end{array}$$

We proceed by re-dividing the remainder by 9, carrying over this Quotient to the quotient side and retaining the final remainder in the remainder side.

$$\begin{array}{rrcl} 9) & 3 & / & 6 \\ & & / & 3 \\ \cline{2-4} & 3 & / & 9 \\ \cline{2-4} & 4 & / & 0 \end{array} \qquad\qquad \begin{array}{rrcl} 9) & 24 & / & 6 \\ & 2 & / & 6 \\ \cline{2-4} & 26 & / & 12 \\ \cline{2-4} & 27 & / & 3 \end{array}$$

Q = 4, R = 0 Q = 27, R = 3.

When the remainder is greater than divisor, it can also be represented as:

9)

$$\begin{array}{rcl} 24 & / & 6 \\ 2 & / & 6 \\ \hline 26 & /1\,/ & 2 \\ & / & 1 \\ \hline 1 & / & 3 \\ \hline 27 & / & 3 \end{array}$$

Now consider the divisors of two or more digits whose last digit is 9, when divisor is 89.

We Know

113 = 1 × 89 + 24, Q = 1, R = 24

10015 = 112 × 89 + 47, Q = 112, R = 47.

Representing in the previous form of procedure, we have

$$\begin{array}{rrcl} 89) & 1 & / & 13 \\ & & / & 11 \\ \hline & 1 & / & 24 \end{array} \qquad \begin{array}{rrcl} 89) & 100 & / & 15 \\ & 12 & / & 32 \\ \hline & 112 & / & 47 \end{array}$$

But how to get these? What is the procedure?

Now Nikhilam rule comes to rescue us. The nikhilam states "all from 9 and the last from 10". Now if you want to find 113 ÷ 89, 10015 ÷ 89, you have to apply nikhilam formula on 89 and get the complement 11. Further while carrying the added numbers to the place below the next digit, we have to multiply by this 11.

$$\begin{array}{rrcl} 89) & 1 & / & 13 \\ & & / & 11 \\ \hline & 1 & / & 24 \end{array} \qquad \begin{array}{rrcll} 89) & 100 & / & 15 & \\ & 11 & / & 11 & \text{first digit } 1 \times 11 \\ & 1 & / & 1 & \text{total second is } 1 \times 11 \\ & & & 22 & \text{total of 3rd digit is } 2 \times 11 \\ \hline & 112 & / & 47 & \end{array}$$

What is 10015 ÷ 98 ? Apply Nikhilam and get 100 - 98 = 02. Set off the 2 digits from the right as the remainder consists of 2 digits. While carrying the added numbers to the place below the next digit, multiply by 02. Thus

$$\begin{array}{llcl}
\underline{98)} & 100 & / & 15 \\
02 & 02 & / & \\
 & 0 & / & 0 \\
 & & / & 04 \\
\hline
 & 102 & / & 19
\end{array}$$

i.e., 10015 ÷ 98 gives Q=102, R=19

In the same way

$$\begin{array}{llcl}
\underline{897)} & 11 & / & 422 \\
103 & 1 & / & 03 \\
 & & / & 206 \\
\hline
 & 12 & / & 658
\end{array}$$

gives 11,422 ÷ 897, Q = 12, R=658.

In this way we have to multiply the quotient by 2 in the case of 8, by 3 in the case of 7, by 4 in the case of 6 and so on. i.e., multiply the Quotient digit by the divisors complement from 10. In case of more digited numbers we apply Nikhilam and proceed. Any how, this method is highly useful and effective for division when the numbers are near to bases of 10.

*** Guess the logic in the process of division by 9.**

*** Obtain the Quotient and Remainder for the following problems.**

(1) 311 ÷ 9

(2) 120012 ÷ 9

(3) 1135 ÷ 97

(4) 2342 ÷ 98

(5) 113401 ÷ 997

(6) 11199171 ÷ 99979

Observe that by nikhilam process of division, even lengthier divisions involve no division or no subtraction but only a few multiplications of single digits with small numbers and a simple addition. But we know fairly well that only a special type of cases are being dealt and hence many questions about various other types of problems arise. The answer lies in Vedic Methods.

Urdhva Tiryagbhyam

Urdhva - tiryagbhyam is the general formula applicable to all cases of multiplication and also in the division of a large number by another large number. It means "Vertically and cross wise."

(a) Multiplication of two 2 digit numbers.

EXAMPLE 1

Find the product 14 × 12

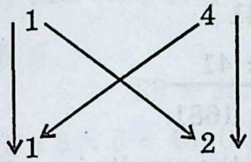
(i) The right hand most digit of the multiplicand, the first number (14) i.e., 4 is multiplied by the right hand most digit of the multiplier, the second number (12) i.e., 2. The product 4 × 2 = 8 forms the right hand most part of the answer.

(ii) Now, diagonally multiply the first digit of the multiplicand (14) i.e., 4 and second digit of the multiplier (12) i.e., 1 (answer 4 × 1 = 4); then multiply the second digit of the multiplicand i.e., 1 and first digit of the multiplier i.e., 2 (answer 1 × 2 = 2); add these two i.e., 4 + 2 = 6. It gives the next, i.e., second digit of the answer. Hence second digit of the answer is 6.

(iii) Now, multiply the second digit of the multiplicand i.e., 1 and second digit of the multiplier i.e., 1 vertically, i.e., 1 × 1 = 1. It gives the left hand most part of the answer.

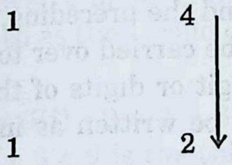
Thus the answer is 16 8.

Symbolically we can represent the process as follows :



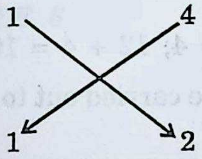
The symbols are operated from right to left .

Step (i) :



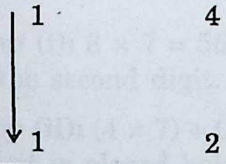
4 × 2

Step (ii) :



2 + 4 : 8

Step (iii) :



1 × 1 : 6 : 8

which gives 168

Now in the same process, answer can be written as

23

13

2 : 6 + 3 : 9 = 299 (Recall the 3 steps)

EXAMPLE 3

$$\begin{array}{r} 41 \\ \times\ 41 \\ \hline 16 : 4+4 : 1 = 1681. \end{array}$$

What happens when one of the results i.e., either in the last digit or in the middle digit of the result, contains more than 1 digit ? Answer is simple. The right - hand - most digit there of is to be put down there and the preceding, i.e., left -hand -side digit or digits should be carried over to the left and placed under the previous digit or digits of the upper row. The digits carried over may be written as in Ex. 4.

EXAMPLE 4

32 × 24

Step (i) : 2 × 4 = 8

Step (ii) : 3 × 4 = 12; 2 × 2 = 4; 12 + 4 = 16.

Here 6 is to be retained. 1 is to be carried out to left side.

Step (iii) : 3 × 2 = 6.

Now the carried over digit 1 of 16 is to be added.

i.e., 6 + 1 = 7.

Thus 32 × 24 = 768

We can write it as follows

$$\begin{array}{r} 32 \\ 24 \\ \hline 668 \\ 1 \\ \hline 768. \end{array}$$

Note that the carried over digit from the result (3×4) + (2 × 2) = 12 + 4 = 16 i.e., 1 is placed under the previous digit 3 × 2 = 6 and added.

After sufficient practice, you feel no necessity of writing in this way and simply operate or perform mentally.

EXAMPLE 5

28 × 35.

Step (i) : 8 × 5 = 40. 0 is retained as the first digit of the answer and 4 is carried over.

Step (ii) : 2 × 5 = 10; 8 × 3 = 24; 10 + 24 = 34; add the carried over 4 to 34. Now the result is 34 + 4 = 38. Now 8 is retained as the second digit of the answer and 3 is carried over.

Step (iii) : 2 × 3 = 6; add the carried over 3 to 6. The result 6 + 3 = 9 is the third or final digit from right to left of the answer.

Thus 28 × 35 = 980.

EXAMPLE 6

$$\begin{array}{r} 48 \\ \underline{47} \\ 1606 \\ \underline{65} \\ 2256 \end{array}$$

Step (i): 8 × 7 = 56; 5, the carried over digit is placed below the second digit.

Step (ii): (4 × 7) + (8 × 4) = 28 + 32 = 60; 6, the carried over digit is placed below the third digit.

Step (iii): Respective digits are added.

ALGEBRAIC PROOF :

(a) Let the two 2 digit numbers be (ax+b) and (cx+d). Note that x = 10. Now consider the product

$$\begin{aligned}(ax + b)(cx + d) &= ac.x^2 + adx + bcx + b.d \\ &= ac.x^2 + (ad + bc)x + b.d\end{aligned}$$

Observe that:

(i) The first term i.e., the coefficient of x^2 (i.e., 100, hence the digit in the 100th place) is obtained by vertical multiplication of a and c i.e., the digits in 10th place (coefficient of x) of both the numbers;

(ii) The middle term, i.e., the coefficient of x (i.e., digit in the 10th place) is obtained by cross wise multiplication of a and d; and of b and c; and the addition of the two products;

(iii) The last (independent of x) term is obtained by vertical multiplication of the independent terms b and d.

(b) Consider the multiplication of two 3 digit numbers.

Let the two numbers be $(ax^2 + bx + c)$ and $(dx^2 + ex + f)$. Note that x=10

Now the product is

$$\begin{array}{r} ax^2 + bx + c \\ \times\ dx^2 + ex + f \\ \hline \end{array}$$

$$ad.x^4+bd.x^3+cd.x^2+ae.x^3+be.x^2+ce.x+af.x^2+bf.x+cf$$

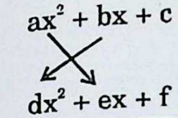
$$= ad.x^4 + (bd + ae).\ x^3 + (cd + be + af).x^2 + (ce + bf)x + cf$$

Note the following points :

(i) The coefficient of x4 , i.e., ad is obtained by the vertical multiplication of the first coefficient from the left side:

$$\begin{array}{l} \big\downarrow\ ax^2 + bx + c \\ \ \ \ dx^2 + ex + f \\ \hline adx^4 \end{array}$$

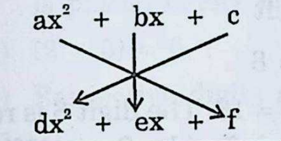
(ii) The coefficient of x3 , i.e., (ae + bd) is obtained by the crosswise multiplication of the first two coefficients and by the addition of the two products;



$aex^3 + bdx^3$

$= (ae + bd)x^3$

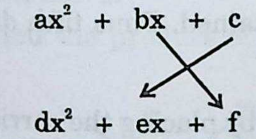
(iii) The coefficient of x^2 is obtained by the multiplication of the first coefficient of the multiplicand $(ax^2 + bx + c)$ i.e., a; by the last coefficient of the multiplier $(dx^2 + ex + f)$ i.e., f ; of the middle one i.e., b of the multiplicand by the middle one i.e., e of the multiplier and of the last one i.e., c of the multiplicand by the first one i.e., d of the multiplier and by the addition of all the three products i.e., af + be +cd :



$afx^2 + bex^2 + cdx^2$

$= (af + be + cd)x^2$

(iv) The coefficient of x is obtained by the cross wise multiplication of the second coefficient i.e., b of the multiplicand by the third one i.e., f of the multiplier, and conversely the third coefficient i.e., c of the multiplicand by the second coefficient i.e., e of the multiplier and by addition of the two products, i.e., bf + ce;



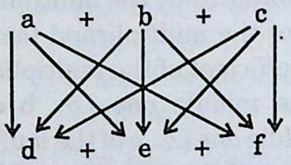
$bfx + cex$

$=(bf + ce)\,x$

(v) And finally the last (independent of x) term is obtained by the vertical multiplication of the last coefficients c and f i.e., cf

$$\begin{array}{r} ax^2 + bx + c\big\downarrow \\ dx^2 + ex + f\,\downarrow \\ \hline cf \qquad \end{array}$$

Thus the process can be put symbolically as (from left to right)



Consider the following example

124 × 132.

Proceeding from right to left

(i) 4 × 2 = 8. First digit = 8

(ii) (2 × 2) + (3 × 4) = 4 + 12 = 16. The digit 6 is retained and 1 is carried over to left side. Second digit = 6.

(iii) (1 × 2) + (2 × 3) + (1 × 4) = 2 + 6 + 4 =12. The carried over 1 of above step is added i.e., 12 + 1 = 13. Now 3 is retained and 1 is carried over to left side. Thus third digit = 3.

(iv) (1 × 3) + (2 × 1) = 3 + 2 = 5. the carried over 1 of above step is added i.e., 5 + 1 = 6 . It is retained. Thus fourth digit = 6

(v) (1 × 1) = 1. As there is no carried over number from the previous step it is retained. Thus fifth digit = 1

124 × 132 = 16368.

Let us work another problem by placing the carried over digits under the first row and proceed.

$$\begin{array}{r} 234 \\ \times\ 316 \\ \hline \end{array}$$

$$\begin{array}{r} 61724 \\ 1222 \\ \hline 73944 \end{array}$$

(i) 4 × 6 = 24 : 2, the carried over digit is placed below the second digit.

(ii) (3 × 6) + (4 x 1) = 18 + 4 = 22 ; 2, the carried over digit is placed below third digit.

(iii) (2 × 6) + (3 × 1) + (4 × 3) = 12 + 3 + 12 = 27 ; 2, the carried over digit is placed below fourth digit.

(iv) (2 × 1) + (3 × 3) = 2 + 9 = 11; 1, the carried over digit is placed below fifth digit.

(v) (2 × 3) = 6.

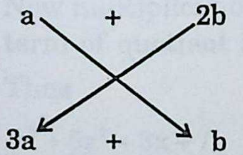
(vi) Respective digits are added.

Note:

1. We can carry out the multiplication in urdhva - tiryak process from left to right or right to left.
2. The same process can be applied even for numbers having more digits.
3. urdhva -tiryak process of multiplication can be effectively used in multiplication regarding algebraic expressions.

EXAMPLE 1:

Find the product of (a + 2b) and (3a + b).



$2a^2 + 7ab + 2b^2$

EXAMPLE 2 :

$$\begin{array}{r} 3a^2 + 2a + 4 \\ \times\ 2a^2 + 5a + 3 \\ \hline \end{array}$$

(i) $4 \times 3 = 12$

(ii) $(2 \times 3) + (4 \times 5) = 6 + 20 = 26$ i.e., $26a$

(iii) $(3 \times 3) + (2 \times 5) + (4 \times 2) = 9 + 10 + 8 = 27$ i.e., $27a^2$

(iv) $(3 \times 5) + (2 \times 2) = 15 + 4 = 19$ i.e., $19\ a^3$

(v) $3 \times 2 = 6$ i.e., $6a^4$

Hence the product is $6a^4 + 19a^3 + 27a^2 + 26a + 12$

EXAMPLE 3 :

Find $(3x^2 + 4x + 7)\ (5x + 6)$

Now $\begin{array}{r} 3.x^2 + 4x + 7 \\ 0.x^2 + 5x + 6 \\ \hline \end{array}$

(i) $7 \times 6 = 42$

(ii) $(4 \times 6) + (7 \times 5) = 24 + 35 = 59$ i.e., $59x$

(iii) $(3 \times 6) + (4 \times 5) + (7 \times 0) = 18 + 20 + 0 = 38$ i.e., $38x^2$

(iv) $(3 \times 5) + (0 \times 4) = 15 + 0 = 15$ i.e., $15x^3$

(v) $3 \times 0 = 0$

Hence the product is $15x^3 + 38x^2 + 59x + 42$

Find the products using urdhva tiryagbhyam process.

(1) 25×16

(2) 32×48

(3) 56×56

(4) 137×214

(5) 321×213

(6) 452 × 348

(7) $(2x + 3y)(4x + 5y)$

(8) $(5a^2 + 1)(3a^2 + 4)$

(9) $(6x^2 + 5x + 2)(3x^2 + 4x + 7)$

(10) $(4x^2 + 3)(5x + 6)$

Urdhva - tiryak in converse for division process:

As per the statement it an used as a simple argumentation for division process particularly in algebra.

Consider the division of $(x^3 + 5x^2 + 3x + 7)$ by $(x - 2)$ process by converse of urdhva - tiryak :

(i) x^3 divided by x gives

$$\frac{x^2.x^3 + 5x^2 + 3x + 7}{x - 2}$$

$Q = x^2 + - - - - - - - - - - -$

It is the first term of the Quotient.

(ii) $x^2 \times -2 = -2x^2$. But $5x^2$ in the dividend hints $7x^2$ more since 7x2 - 2x2 = 5x2 . This 'more' can be obtained from the multiplication of x by 7x. Hence second term of Q is 7x.

$$\frac{x^3 + 5x^2 + 3x + 7}{x - 2}$$

gives Q = x2 + 7x + - - - - - - - -

(iii) We now have - 2 × 7x = -14x. But the 3rd term in the dividend is 3x for which '17x more' is required since 17x - 14x =3x.

Now multiplication of x by 17 gives 17x. Hence third term of quotient is 17

Thus

$$\frac{x^3 + 5x^2 + 3x + 7}{x - 2}$$

gives Q= x2 + 7x +17

(iv) Now last term of Q, i.e., 17 multiplied by -2 gives 17 × -2 = -34 but the relevant term in dividend is 7. So 7 + 34 = 41 'more' is required. As there no more terms left in dividend, 41 remains as the remainder.

$$\frac{x^3+5x^2+3x+7}{x-2}$$

gives Q= $x^2 + 7x + 17$ and R = 41.

Find the Q and R in the following divisions by using the converse process of urdhva - tiryagbhyam method:

1. $\dfrac{3x^2-x-6}{3x-7}$

2. $\dfrac{16x^2+24x+9}{4x+3}$

3. $\dfrac{x^3+2x^2+3x+5}{x-3}$

4. $\dfrac{12x^4-3x^2-3x+12}{x^2+1}$

Paravartya—Yojayet

'Paravartya - Yojayet' means 'transpose and apply'

(i) Consider the division by divisors of more than one digit, and when the divisors are slightly greater than powers of 10.

EXAMPLE 1 :

Divide 1225 by 12.

Step 1 : (From left to right) write the Divisor leaving the first digit, write the other digit or digits using negative (-) sign and place them below the divisor as shown.

12
-2

Step 2 : Write down the dividend to the right. Set apart the last digit for the remainder.

i.e., 12 122 5
- 2

Step 3 : Write the 1st digit below the horizontal line drawn under the dividend. Multiply the digit by -2, write the product below the 2nd digit and add.

i.e., 12 122 5
-2 -2
10

Since 1 × -2 = -2 and 2 + (-2) = 0

Step 4 : We get second digits' sum as '0'. Multiply the second digits' sum thus obtained by -2 and writes the product under 3rd digit and add.

12 122 5
- 2 -20
102 5

Step 5 : Continue the process to the last digit.

i.e., 12 122 5
- 2 -20 -4
102 1

Step 6: The sum of the last digit is the Remainder and the result to its left is Quotient.

Thus Q = 102 and R = 1

EXAMPLE 2 :

Divide 1697 by 14.

$$\begin{array}{c|cccc} 14 & 1 & 6 & 9 & 7 \\ \underline{-4} & & -4 & -8 & -4 \\ \hline & 1 & 2 & 1 & 3 \end{array}$$

Q = 121, R = 3.

EXAMPLE 3 :

Divide 2598 by 123.

Note that the divisor has 3 digits. So we have to set up the last two digits of the dividend for the remainder.

$$\begin{array}{lr} 1\ 2\ 3 & 25\ 98 \text{ Step (1) \& Step (2)} \\ \quad -2-3 & \\ \hline & \end{array}$$

Now proceed the sequence of steps write -2 and -3 as follows :

$$\begin{array}{cccc} 1\ 2\ 3 & 2\ 5 & & 9\ 8 \\ -2-3 & -4 & & -6 \\ \hline & & & -2-3 \\ \hline & 2\ 1 & & 1\ 5 \end{array}$$

Since2 × (-2, -3)= -4 , -6; 5 - 4 = 1

and (1 × (-2,-3); 9 - 6 - 2 = 1; 8 - 3 = 5.

Hence Q = 21 and R = 15.

EXAMPLE 4 :

Divide 239479 by 11213. The divisor has 5 digits. So the last 4 digits of the dividend are to be set up for Remainder.

1 1 2 1 3	2 3	9 4 7 9	
-1-2-1-3	-2	-4-2-6	with 2
		-1-2-1-3	with 1
	2 1	4 0 0 6	

Hence Q = 21, R = 4006.

EXAMPLE 5 :

Divide 13456 by 1123

1 1 2 3	1 3 4 5	6
-1-2-3	-1-2-3	
	-2-4	-6
	1 2 0-2	0

Note that the remainder portion contains -20, i.e., a negative quantity. To over come this situation, take 1 over from the quotient column, i.e., 1123 over to the right side, subtract the remainder portion 20 to get the actual remainder.

Thus Q = 12 - 1 = 11,

and R = 1123 - 20 = 1103.

Find the Quotient and Remainder for the problems using paravartya - yojayet method.

(1) 1234 ÷ 112

(2) 11329 ÷ 1132

(3) 12349 ÷ 133

(4) 239479 ÷ 1203

Now let us consider the application of paravartya - yojayet in algebra.

EXAMPLE 1 :

Divide $6x^2 + 5x + 4$ by $x - 1$

$$\begin{array}{l r}
\underline{X - 1} & 6x^2 + 5x + 4 \\
1 & \underline{\quad 6 + 11} \\
 & 6x + 11 + 15
\end{array}$$

Thus Q = 6x+11, R=15.

EXAMPLE 2 :

Divide $x^3 - 3x^2 + 10x - 4$ by $x - 5$

$$\begin{array}{l r}
\underline{X - 5} & x^3 - 3x^2 + 10x - 4 \\
5 & \underline{\quad 5 + 10 \quad 100} \\
 & x^2 + 2x + 20, + 96
\end{array}$$

Thus $Q= x^2 + 2x + 20$, $R = 96$.

The procedure as a mental exercise comes as follows :

(i) x^3 / x gives x^2 i.e., 1 the first coefficient in the Quotient.

(ii) Multiply 1 by + 5, (obtained after reversing the sign of second term in the Quotient) and add to the next coefficient in the dividend. It gives $1 \times (+5) = +5$, adding to the next coefficient, i.e., $-3 + 5 = 2$. This is next coefficient in Quotient.

(iii) Continue the process : multiply 2 by +5, i.e., $2 \times +5 = 10$, add to the next coefficient $10 + 10 = 20$. This is next coefficient in Quotient. Thus Quotient is $x^2 + 2x + 20$

(iv) Now multiply 20 by + 5 i.e., $20 \times 5 = 100$. Add to the next (last) term, $100 + (-4) = 96$, which becomes R, i.e., R = 9.

EXAMPLE 3:

$$\frac{x^4 - 3x^3 + 7x^2 + 5x + 7}{x + 4}$$

Now thinking the method as in example (1), we proceed as follows.

$$\begin{array}{lr}
\underline{x + 4} & x^4 - 3x^3 + 7x^2 + 5x + \quad 7 \\
 & \\
 & \underline{-4 - 4 + 28 - 140 + 540} \\
 & \\
 & x^3 - 7x2 + 35x - 135 \quad 547
\end{array}$$

Thus $Q = x^3 - 7x^2 + 35x - 135$ and $R = 547$.

or we proceed orally as follows:

x^4/x gives 1 as first coefficient.

(i) $-4 \times 1 = -4$: add to next coefficient $-4 + (-3) = -7$ which gives next coefficient in Q.

(ii) $-7 \times -4 = 28$: then $28 + 7 = 35$, the next coefficient. in Q.

(iii) $35 \times -4 = -140$: then $-140 + 5 = -135$, the next coefficient in Q.

(iv) $-135 \times -4 = 540$: then $540 + 7 = 547$ becomes R.

Thus $Q = x3 - 7x2 + 35x - 135$, $R = 547$.

Note :

1. We can follow the same procedure even the number of terms is more.
2. If any term is missing, we have to take the coefficient of the term as zero and proceed.

Now consider the divisors of second degree or more as in the following example.

EXAMPLE :4

$2x^4 - 3x^3 - 3x + 2$ by $x^2 + 1$.

Here x^2 term is missing in the dividend. Hence treat it as $0 . x^2$ or 0 . And the x term in divisor is also absent we treat it as $0 . x$. Now

$$\begin{array}{l|ccccc}
x^2 \qquad + 1 & 2x^4 & -\ 3x^3 & +\ 0\ .\ x^2 & -\ 3x & +\ 2 \\
\underline{x^2 + 0\ .\ x + 1} & & 0 & -2 & & \\
 & & & & & \\
0 \qquad -1 & & & 0 & +3 & \\
 & & & & 0 & +2 \\
\hline
 & 2 & -3 & -2 & 0 & 4
\end{array}$$

Thus $Q = 2x^2 - 3x - 2$ and $R = 0\ .\ x + 4 = 4$.

EXAMPLE 5 :

$2x^5 - 5x^4 + 3x^2 - 4x + 7$ by $x^3 - 2x^2 + 3$.

We treat the dividend as $2x^5 - 5x^4 + 0.\ x^3 + 3x^2 - 4x + 7$ and divisor as $x^3 - 2x^2 + 0\ .\ x + 3$ and proceed as follows:

$$\begin{array}{l|cccccc}
\underline{x^3 - 2x^2 + 0\ .\ x + 3} & 2x^5 & -\ 5x^4 & +\ 0.x^3 & +\ 3x^2 & -\ 4x & +\ 7 \\
2 \qquad 0 \qquad -3 & & 4 & 0 & -6 & & \\
 & & & -2 & 0 & +3 & \\
 & & & & -4 & 0 & +6 \\
\hline
 & 2 & -1 & -2 & -7 & -1 & +13
\end{array}$$

Thus $Q = 2x^2 - x - 2$, $R = - 7x^2 - x + 13$.

You may observe a very close relation of the method paravartya in this aspect with regard to REMAINDER THEOREM and HORNER PROCESS of Synthetic division. And yet paravartya goes much farther and is capable of numerous applications in other directions also.

Apply paravartya - yojayet to find out the Quotient and Remainder in each of the following problems.

(1) $(4x^2 + 3x + 5) \div (x+1)$

(2) $(x^3 - 4x^2 + 7x + 6) \div (x - 2)$

(3) $(x^4 - x^3 + x^2 + 2x + 4) \div (x2 - x - 1)$

(4) $(2x^5 + x^3 - 3x + 7) \div (x3 + 2x - 3)$

(5) $(7x^6 + 6x^5 - 5x^4 + 4x^3 - 3x^2 + 2x - 1) \div (x-1)$

Paravartya in solving simple equations :

Recall that 'paravartya yojayet' means 'transpose and apply'. The rule relating to transposition enjoins invariable change of sign with every change of side. i.e., + becomes - and conversely; and X becomes ÷ and conversely. Further it can be extended to the transposition of terms from left to right and conversely and from numerator to denominator and conversely in the concerned problems.

Type (i) :

Consider the problem

$$7x - 5 = 5x + 1$$

$$7x - 5x = 1 + 5$$

i.e.,$2x = 6$ $x = 6 \div 2 = 3$.

Observe that the problem is of the type $ax + b = cx + d$ from which we get by 'transpose' $(d - b)$, $(a - c)$ and

$$x = \frac{d-b}{a-c}$$

In this example $a = 7$, $b = -5$, $c = 5$, $d = 1$

Hence

$$x = \frac{1-(-5)}{7-5} = \frac{1+5}{7-5} = \frac{6}{2} = 3$$

EXAMPLE 2:

Solve for x, $3x + 4 = 2x + 6$

$$x = \frac{d-b}{a-c} = \frac{6-4}{3-2} = \frac{2}{1} = 1$$

Type (ii):

Consider problems of the type $(x + a)(x+b) = (x+c)(x+d)$. By paravartya, we get

$$x = \frac{cd - ab}{(a+b)-(c+d)}$$

It is trivial form the following steps

$(x + a)(x + b) = (x + c)(x + d)$

$x^2 + bx + ax + ab = x^2 + dx + cx + cd$

$bx + ax - dx - cx = cd - ab$

$x(a + b - c - d) = cd - ab$

$$x = \frac{cd - ab}{a+b-c-d} \quad x = \frac{cd - ab}{(a+b)-(c+d)}$$

EXAMPLE 1 :

$(x - 3)(x - 2) = (x + 1)(x + 2)$.

By paravartya

$$x = \frac{cd - ab}{a+b-c-d} = \frac{1(2)-(-3)(-2)}{-3-2-1-1}$$

$$= \frac{2-6}{-8} = \frac{-4}{-8} = \frac{1}{2}$$

EXAMPLE 2 :

$(x + 7)(x - 6) = (x + 3)(x - 4)$.

Now

$$x = \frac{cd - ab}{a+b-c-d} = \frac{(3)(-4)-(7)(-6)}{7+(-6)-3-(-4)}$$

$$= \frac{-12+42}{7-6-3+4} = \frac{30}{2} = 15$$

Note that if $cd - ab = 0$ i.e., $cd = ab$, i.e., if the product of the absolute terms be the same on both sides, the numerator becomes zero giving $x = 0$.

For the problem $(x + 4)(x + 3) = (x - 2)(x - 6)$

Solution is $x = 0$ since $4 \times 3 = -2 \times -6. = 12$

Type (iii) :

Consider the problems of the type

$$\frac{ax+b}{cx+d} = \frac{m}{n}$$

By cross - multiplication,

$$n(ax + b) = m(cx + d)$$

$$nax + nb = mcx + md$$

$$nax - mcx = md - nb$$

$$x(na - mc) = md - nb$$

$$x = \frac{md-nb}{na-mc}.$$

Now look at the problem once again

$$\frac{ax+b}{cx+d} = \frac{m}{n}$$

paravartya gives md - nb, na - mc and

$$x = \frac{md-nb}{na-mc}.$$

EXAMPLE 1:

$$\frac{3x+1}{4x+3} = \frac{13}{19}$$

$$x = \frac{md-nb}{na-mc} = \frac{13(3)-19(1)}{19(3)-13(4)}$$

$$= \frac{39-19}{57-52} = \frac{20}{5} = 4.$$

EXAMPLE 2:

$$\frac{4x+5}{3x+\frac{13}{2}}=\frac{7}{8}$$

$$x=\frac{(7)\left(\frac{13}{2}\right)-(8)(5)}{(8)(4)-(7)(3)}=\frac{\left(\frac{91}{2}\right)-40}{32-21}$$

$$=\frac{\frac{(91-80)}{2}}{32-21}=\frac{11}{2\times 11}=\frac{1}{2}.$$

Type (iv) :

Consider the problems of the type

$$\frac{m}{x+a}+\frac{n}{x+b}=0$$

Take L.C.M and proceed.

$$\frac{m(x+b)+n(x+a)}{(x+a)(x+b)}=0$$

$$\frac{mx+mb+nx+na}{(x+a)(x+b)}=0$$

$$(m + n)x + mb + na = 0$$

$$\therefore \quad (m + n)x = -mb - na$$

$$x=\frac{-mb-na}{(m+n)}.$$

Thus the problem $\frac{m}{x+a}+\frac{n}{x+b}=0$, by paravartya process gives directly

$$x=\frac{-mb-na}{(m+n)}.$$

EXAMPLE 1 :

$$\frac{3}{x+4}+\frac{4}{x-6}=0$$

gives

$$x=\frac{-mb-na}{(m+n)}.$$

Note that m = 3, n = 4, a = 4, b = - 6

$$=\frac{-(3)(-6)-(4)(4)}{(3+4)}=\frac{18-16}{7}=\frac{2}{7}.$$

EXAMPLE 2 :

$$\frac{5}{x+1}+\frac{6}{x-21}=0$$

gives

$$x=\frac{-(5)(-21)-(6)(1)}{5+6}$$

$$=\frac{105-6}{11}=\frac{99}{11}=9$$

I. Solve the following problems using the sutra Paravartya - yojayet.

(1) $3x + 5 = 5x - 3$

(2) $(2x/3) + 1 = x - 1$

(3) $\frac{7x+2}{3x-5}=\frac{5}{8}$

(4) $\frac{x+1/3}{3x-1}=1$

(5) $\frac{5}{x+3}+\frac{2}{x-4}=0$

(6) $(x + 1)(x + 2) = (x - 3)(x - 4)$

(7) $(x - 7)(x - 9) = (x - 3)(x - 22)$

(8) $(x + 7)(x + 9) = (x + 3)(x + 21)$

(II)

1. Show that for the type of equations

$$\frac{m}{x+a}+\frac{n}{x+b}+\frac{p}{x+c}=0, \text{ the solution is}$$

$$x=\frac{-mbc-nca-pab}{m(b+c)+n(c+a)+p(a+b)}, \text{ if } m+n+p=0.$$

2. Apply the above formula to set the solution for the problem

$$\frac{3}{x+4}+\frac{2}{x+6}-\frac{5}{x+5}=0.$$

Some more simple solutions :

$$\frac{m}{x+a}+\frac{n}{x+b}+\frac{m+n}{x+c}$$

Now this can be written as,

$$\frac{m}{x+a}+\frac{n}{x+b}=\frac{m}{x+c}+\frac{n}{x+c}$$

$$\frac{m}{x+a}-\frac{m}{x+c}=\frac{n}{x+c}-\frac{n}{x+b}$$

$$\frac{m(x+c)-m(x+a)}{(x+a)(x+c)}=\frac{n(x+b)-n(x+c)}{(x+c)(x+b)}$$

$$\frac{mx+mc-mx-ma}{(x+a)(x+c)}=\frac{nx+nb-nx-nc}{(x+c)(x+b)}$$

$$\frac{m(c-a)}{x+a} = \frac{n(b-c)}{x+b}$$

m (c - a).x + m (c - a).b = n (b - c). x + n(b - c).a

x [m(c - a) - n(b - c)] = na(b - c) - mb (c - a)

orx [m(c - a) + n(c - b)]= na(b - c) + mb (a - c)

Thus

$$x = \frac{mb(a-c)+na(b-c)}{m(c-a)+n(c-b)}.$$

By paravartya rule we can easily remember the formula.

EXAMPLE 1 :

Solve $\frac{3}{x+1}+\frac{4}{x+2}=\frac{7}{x+3}$

In the usual procedure, we proceed as follows.

$$\frac{3}{x+1}+\frac{4}{x+2} = \frac{7}{x+3}$$

$$\frac{3(x+2)+4(x+1)}{(x+1)(x+2)} = \frac{7}{x+3}$$

$$\frac{3x+6+4x+4}{x^2+2x+x+2} = \frac{7}{x+3}$$

$$(7x + 10)\ (x + 3) = 7(x^2 + 3x + 2)$$

$$7x^2 + 21x + 10x + 30 = 7x^2 + 21x + 14.$$

$$31x + 30 = 21x + 14$$

$$31x - 21x = 14 - 30$$

i.e., $10x = - 16$

$$x = - 16 / 10 = - 8 / 5$$

Now by Paravartya process

$$\frac{3}{x+1}+\frac{4}{x+2} = \frac{7}{x+3}$$

$$(\ldots\ N_1 + N_2 = 3+4 = 7 = N_3)$$

$$x = \frac{mb(a-c)+na(b-c)}{m(c-a)+n(c-b)}$$

here $N_1 = m = 3$, $N_2 = n = 4$; $a = 1$, $b = 2$, $c = 3$

$$= \frac{3.2(1-3)+4.1.(2-3)}{3(3-1)+4(3-2)}$$

$$= \frac{6(-2)+4(-1)}{3(2)+4(1)} = \frac{-12-4}{6+4}$$

$$= \frac{-16}{10} = \frac{-8}{5}$$

EXAMPLE 2 :

$$\frac{3}{x-2}+\frac{5}{x-6} = \frac{8}{x+3}$$

Here $N_1 + N_2 = 3 + 5 = 8$.

$$x = \frac{mb(a-c)+na(b-c)}{m(c-a)+n(c-b)}$$

$$= \frac{3.(-6)(-2-3)+5.(-2)(6-3)}{3(3-(-2))+5(-3(-6))}$$

$$= \frac{3(-6)(-5)+5(-2)(-9)}{3(3+2)+5(3+6)}$$

$$= \frac{90+90}{15+45} = \frac{180}{60} = 3.$$

Solve the problems using the methods explained above.

(1) $\frac{2}{x+2}+\frac{3}{x+3}=\frac{5}{x+5}$

(2) $\frac{4}{x+1}+\frac{6}{x+3}=\frac{10}{x+4}$

(3) $\frac{5}{x-2}+\frac{2}{3-x}=\frac{3}{x-4}$

(4) $\frac{4}{2x+1}+\frac{9}{3x+2}=\frac{15}{3x+1}$

Note : The problem (4) appears to be not in the model said above.

But $\frac{3(4)}{3(2x+1)}+\frac{2(9)}{2(3x+2)}=\frac{2(15)}{2(3x+1)}$ gives

$$\frac{12}{6x+3}+\frac{18}{6x+4}=\frac{30}{6x+2}.$$

Now proceed.

Simultaneous Simple Equations

By applying Paravartya sutra we can derive the values of x and y which are given by two simultaneous equations. The values of x and y are given by ration form. The method to find out the numerator and denominator of the ratio is given below.

EXAMPLE 1:

$2x + 3y = 13,\ 4x + 5y = 23.$

(i) To get x, start with y coefficients and the independent terms and cross-multiply forward, i.e., right ward. Start from the upper row and multiply across by the lower one, and conversely, the connecting link between the two cross-products being a minus. This becomes numerator.

i.e., $2x + 3y = 13$

$4x + 5y = 23$

Numerator of the x - value is $3 \times 23 - 5 \times 13 = 69 - 65 = 4$

(ii) Go from the upper row across to the lower one, i.e., the x- coefficient but backward, i.e., leftward.

Denominator of the x - value is $3 \times 4 - 2 \times 5 = 12 - 10 = 2$

Hence value of $x = 4 \div 2 = 2$.

(iii) To get y, follow the cyclic system, i.e., start with the independent term on the upper row towards the x-coefficient on the lower row. So numerator of the y-value is

$13 \times 4 - 23 \times 2 = 52 - 46 = 6$.

(iv) The denominator is the same as obtained in Step (ii) i.e., 2. Hence value of y is $6 \div 2 = 3$.

Thus the solution to the given equation is $x = 2$ and $y = 3$.

EXAMPLE 2:

$5x - 3y = 11$

$6x - 5y = 09$

NowNr. of x is $(-3)(9) - (5)(11) = -27 + 55 = 28$

Dr. of x is $(-3)(6) - (5)(-5) = -18 + 25 = 07$

$x = \text{Nr} \div \text{Dr.} = 28 \div 7 = 4$

and for y,

Nr is $(11)(6) - (9)(5) = 66 - 45 = 21$

Dr. is 7

Hence $y = 21 \div 7 = 3$.

EXAMPLE 3:

Solve $3x + y = 5$

$4x - y = 9$

Now we can straight away write the values as follows:

$$x = \frac{(1)(9)-(-1)(5)}{(1)(4)-(3)(-1)} = \frac{9+5}{4+3} = \frac{14}{7} = 2$$

$$y = \frac{(5)(4)-(9)(3)}{(1)(4)-(3)(-1)} = \frac{20-27}{4+3} = \frac{-7}{7} = -1.$$

Hence $x = 2$ and $y = -1$ is the solution.

ALGEBRAIC PROOF:

$$ax + by = m \quad \text{.........} \quad \text{(i)}$$

$$cx + dy = n \quad \text{..........} \quad \text{(ii)}$$

Multiply (i) by d and (ii) by b, then subtract

$$adx + bdy = m.d$$

$$cbx + dby = n.b$$

$$(ad - cb) .x = md - nb$$

$$x = \frac{md - nb}{ad - cb} = \frac{bn - md}{bc - ad}$$

Multiply (i) by c and (ii) by a, then subtract

$$acx + bcy = m.c$$

$$cax + day = n.a$$

$$(bc - ad) . y = mc - na$$

$$y = \frac{mc - na}{bc - ad}$$

You feel comfort in the Paravartya process because it avoids the confusion in multiplication, change of sign and such other processes.

Find the values of x and y in each of the following problems using Paravartya process.

1. $2x + y = 5$

   $3x - 4y = 2$

2. $3x - 4y = 7$

   $5x + y = 4$

3. $4x + 3y = 8$

   $6x - y = 1$

4. $x + 3y = 7$

   $2x + 5y = 11$

Sunyam Samyasamuccaye

The Sutra 'Sunyam Samyasamuccaye' says the 'Samuccaya is the same, that Samuccaya is Zero.' i.e., it should be equated to zero. The term 'Samuccaya' has several meanings under different contexts.

(i) We interpret, 'Samuccaya' as a term which occurs as a common factor in all the terms concerned and proceed as follows.

EXAMPLE 1:

The equation $7x + 3x = 4x + 5x$ has the same factor ' x ' in all its terms. Hence by the sutra it is zero, i.e., $x = 0$.

Otherwise we have to work like this:

$$7x + 3x = 4x + 5x$$

$$10x = 9x$$

$$10x - 9x = 0$$

$$x = 0$$

This is applicable not only for 'x' but also any such unknown quantity as follows.

EXAMPLE 2:

$$5(x+1) = 3(x+1)$$

No need to proceed in the usual procedure like

$$5x + 5 = 3x + 3$$

$$5x - 3x = 3 - 5$$

$$2x = -2$$

or $$x = -2 \div 2 = -1$$

Simply think of the contextual meaning of ' Samuccaya'

Now Samuccaya is $(x + 1)$

$$x + 1 = 0 \text{ gives } x = -1$$

(ii) Now we interpret 'Samuccaya 'as product of independent terms in expressions like $(x+a)(x+b)$

EXAMPLE 3:

$$(x + 3)(x + 4) = (x - 2)(x - 6)$$

Here Samuccaya is $3 \times 4 = 12 = -2 \times -6$

Since it is same , we derive $x = 0$

This example, we have already dealt in type (ii) of Paravartya in solving simple equations.

(iii) We interpret ' Samuccaya ' as the sum of the denominators of two fractions having the same numerical numerator.

Consider the example.

$$\frac{1}{3x-2} + \frac{1}{2x-1} = 0$$

for this we proceed by taking L.C.M.

$$\frac{(2x-1)+(3x-2)}{(3x-2)(2x-1)} = 0$$

$$\frac{5x-3}{(3x-2)(2x-1)} = 0$$

$$5x - 3 = 0$$

$$5x = 3$$

$$x = \frac{3}{5}$$

Instead of this, we can directly put the Samuccaya i.e., sum of the denominators

i.e., $3x - 2 + 2x - 1 = 5x - 3 = 0$

giving $5x = 3$

$x = 3 / 5$

It is true and applicable for all problems of the type

$$\frac{m}{ax+b} + \frac{m}{cx+d} = 0$$

Samuccaya is ax+b+cx+d and solution is $(m \neq 0)$

$$x = \frac{-(b+d)}{(a+c)}$$

(iii) We now interpret 'Samuccaya' as combination or total.

If the sum of the numerators and the sum of the denominators be the same, then that sum = 0.

Consider examples of type

$$\frac{ax+b}{ax+c} = \frac{ax+c}{ax+b}$$

In this case, (ax+b) (ax+b) = (ax+c) (ax+c)

$$a^2x^2 + 2abx + b^2 = a^2x^2 + 2acx + c^2$$

$$2abx - 2acx = c^2 - b^2$$

$$x\,(2ab - 2ac) = c^2 - b^2$$

$$x = \frac{c^2 - b^2}{2a(b-c)}$$

$$= \frac{(c+b)(c-b)}{2a(b-c)} = \frac{-(c+b)}{2a}$$

As per Samuccaya

$$(ax+b) + (ax+c) = 0$$

$$2ax+b+c = 0$$

$$2ax = -b-c$$

$$x = \frac{-(c+b)}{2a}$$

Hence the statement.

EXAMPLE 4:

$$\frac{3x+4}{3x+5} = \frac{3x+5}{3x+4}$$

Since $N_1 + N_2 = 3x + 4 + 3x + 5 = 6x + 9$,

And $D_1 + D_2 = 3x + 4 + 3x + 5 = 6x + 9$

We have $N_1 + N_2 = D_1 + D_2 = 6x + 9$

Hence from Sunya Samuccaya we get 6x + 9 = 0

$$6x = -9$$

$$x = \frac{-9}{6} = \frac{-3}{2}$$

EXAMPLE 5:

$$\frac{5x+7}{5x+12}=\frac{5x+12}{5x+7}$$

Hence $N_1 + N_2 = 5x + 7 + 5x + 12 = 10x + 19$

And$D_1 + D_2 = 5x + 12 + 5x + 7 = 10x + 19$

$N_1 + N_2 = D_1 + D_2$ gives $10x + 19 = 0$

$$10x = -19$$

$$x=\frac{-19}{10}.$$

Consider the examples of the type, where $N_1 + N_2 = K (D_1 + D_2)$, where K is a numerical constant, then also by removing the numerical constant K, we can proceed as above.

EXAMPLE 6:

$$\frac{2x+3}{4x+5}=\frac{x+1}{2x+3}$$

Here $N_1 + N_2 = 2x + 3 + x + 1 = 3x + 4$

$D_1 + D_2 = 4x + 5 + 2x + 3 = 6x + 8 = 2 (3x + 4)$

Removing the numerical factor 2, we get 3x + 4 on both sides.

$3x + 4 = 0$ $3x = -4$ $x = - 4/3$.

(v) 'Samuccaya' with the same meaning as above, i.e., case (iv), we solve the problems leading to quadratic equations. In this context, we take the problems as follows;

If $N_1 + N_2 = D_1 + D_2$ and also the differences

$N_1 \sim D_1 = N_2 \sim D_2$ then both the things are equated to zero, the solution gives the two values for x.

EXAMPLE 7:

$$\frac{3x+2}{2x+5} = \frac{2x+5}{3x+2}$$

In the conventional text book method, we work as follows:

$$\frac{3x+2}{2x+5} = \frac{2x+5}{3x+2}$$

$$(3x + 2)(3x + 2) = (2x + 5)(2x + 5)$$

$$9x^2 + 12x + 4 = 4x^2 + 20x + 25$$

$$9x^2 + 12x + 4 - 4x^2 - 20x - 25 = 0$$

$$5x^2 - 8x - 21 = 0$$

$$5x^2 - 15x + 7x - 21 = 0$$

$$5x(x - 3) + 7(x - 3) = 0$$

$$(x - 3)(5x + 7) = 0$$

$$x - 3 = 0 \text{ or } 5x + 7 = 0$$

$$x = 3 \text{ or } -7/5$$

Now 'Samuccaya' sutra comes to help us in a beautiful way as follows :

Observe $N_1 + N_2 = 3x + 2 + 2x + 5 = 5x + 7$

$D_1 + D_2 = 2x + 5 + 3x + 2 = 5x + 7$

Further $N_1 \sim D_1 = (3x + 2) - (2x + 5) = x - 3$

$N_2 \sim D_2 = (2x + 5) - (3x + 2)$

$= -x + 3 = -(x - 3)$

Hence $5x + 7 = 0$, $x - 3 = 0$

$5x = -7$, $x = 3$

i.e., $x = -7/5$, $x = 3$

Note that all these can be easily calculated by mere observation.

EXAMPLE 8:

$$\frac{3x+4}{6x+7} = \frac{5x+6}{2x+3}$$

Observe that

$$N_1 + N_2 = 3x + 4 + 5x + 6 = 8x + 10$$

and $$D_1 + D_2 = 6x + 7 + 2x + 3 = 8x + 10$$

Further $$N_1 \sim D_1 = (3x + 4) - (6x + 7)$$

$$= 3x + 4 - 6x - 7$$

$$= -3x - 3 = -3\ (x + 1)$$

$$N_2 \sim D_2 = (5x + 6) - (2x + 3)$$

$$= 3x + 3 = 3(x + 1)$$

By 'Sunyam Samuccaye' we have

$$8x + 10 = 0 \qquad 3(x + 1) = 0$$

$$8x = -10 \qquad x + 1 = 0$$

$$x = -10 / 8 \qquad x = -1$$

$$= -5 / 4$$

(vi) 'Samuccaya' with the same sense but with a different context and application.

EXAMPLE 9:

$$\frac{1}{x-4}+\frac{1}{x-6} = \frac{1}{x-2}+\frac{1}{x-8}$$

Usually we proceed as follows.

$$\frac{x-6+x-4}{(x-4)(x-6)} = \frac{x-8+x-2}{(x-2)(x-8)}$$

$$\frac{2x-10}{x^2-10x+24} = \frac{2x-10}{x^2-10x+16}$$

(2x - 10) (x2 - 10x + 16) = (2x - 10) (x2 - 10x + 24)

$$2x^3-20x^2+32x-10x^2+100x-160 = 2x^3-20x^2+48-10x^2+100x-240$$

$$2x^3 - 30x^2 + 132x - 160 = 2x^3 - 30x^2 + 148x - 240$$

$$132x - 160 = 148x - 240$$

$$132x - 148x = 160 - 240$$

$$- 16x = - 80$$

$$x = - 80 / - 16 = 5$$

Now 'Samuccaya' sutra, tell us that, if other elements being equal, the sum-total of the denominators on the L.H.S. and their total on the R.H.S. be the same, that total is zero.

Now $D_1 + D_2 = x - 4 + x - 6 = 2x - 10$, and

$D_3 + D_4 = x - 2 + x - 8 = 2x - 10$

By Samuccaya, $2x - 10$ gives $2x = 10$

$$x = \frac{10}{2} = 5.$$

EXAMPLE 10:

$$\frac{1}{x-8}+\frac{1}{x-9} = \frac{1}{x-5}+\frac{1}{x-12}$$

$D_1 + D_2 = x - 8 + x - 9 = 2x - 17$, and

$D_3 + D_4 = x - 5 + x -12 = 2x - 17$

Now $2x - 17 = 0$ gives $2x = 17$

$$x = \frac{17}{2} = 8\frac{1}{2}.$$

EXAMPLE 11:

$$\frac{1}{x+7}-\frac{1}{x+10} = \frac{1}{x+6}-\frac{1}{x+9}$$

This is not in the expected form. But a little work regarding transposition makes the above as follows.

$$\frac{1}{x+7}+\frac{1}{x+10} = \frac{1}{x+6}-\frac{1}{x+9}$$

Now 'Samuccaya' sutra applies

$$D_1 + D_2 = x + 7 + x + 9 = 2x + 16, \text{ and}$$

$$D_3 + D_4 = x + 6 + x + 10 = 2x + 16$$

Solution is given by $2x + 16 = 0$ i.e., $2x = -16$.

$$x = -16 / 2 = -8.$$

Solve the following problems using Sunyam Samya-Samuccaye process.

1. $7 (x + 2) + 3 (x + 2) = 6 (x + 2) + 5 (x + 2)$
2. $(x + 6) (x + 3) = (x - 9) (x - 2)$
3. $(x - 1) (x + 14) = (x + 2) (x - 7)$
4. $\frac{1}{4x-3}+\frac{1}{x-2}=0$
5. $\frac{4}{3x+1}+\frac{4}{5x+7}=0$
6. $\frac{2x+11}{2x+5}=\frac{2x+5}{2x+11}$
7. $\frac{3x+4}{6x+7}=\frac{x+1}{2x+3}$
8. $\frac{4x-3}{2x+3}=\frac{x+4}{3x-2}$
9. $\frac{1}{x-2}+\frac{1}{x-5}=\frac{1}{x-3}+\frac{1}{x-4}$
10. $\frac{1}{x-7}-\frac{1}{x-6}=\frac{1}{x-10}-\frac{1}{x-9}$

Sunyam Samya Samuccaye in Certain Cubes

Consider the problem $(x - 4)^3 + (x - 6)^3 = 2(x - 5)^3$. For the solution by the traditional method we follow the steps as given below:

$$(x - 4)^3 + (x - 6)^3 = 2(x - 5)^3$$

$$x^3 - 12x^2 + 48x - 64 + x^3 - 18x^2 + 108x - 216$$

$$= 2(x^3 - 15x^2 + 75x - 125)$$

$$2x^3 - 30x^2 + 156x - 280 = 2x^3 - 30x^2 + 150x - 250$$

$$156x - 280 = 150x - 250$$

$$156x - 150x = 280 - 250$$

$$6x = 30$$

$$x = 30 / 6 = 5.$$

But once again observe the problem in the vedic sense.

We have $(x - 4) + (x - 6) = 2x - 10$. Taking out the numerical factor 2, we have $(x - 5) = 0$, which is the factor under the cube on R.H.S. In such a case "Sunyam samya Samuccaye" formula gives that $x - 5 = 0$. Hence $x = 5$

Think of solving the problem

$$(x-249)^3 + (x+247)^3 = 2(x-1)^3$$

The traditional method will be horrible even to think of.

But $(x - 249) + (x + 247) = 2x - 2 = 2(x - 1)$. And $x - 1$. on R.H.S. cube, it is enough to state that $x - 1 = 0$ by the 'sutra'.

$x = 1$ is the solution. No cubing or any other mathematical operations.

ALGEBRAIC PROOF :

Consider $(x - 2a)^3 + (x - 2b)^3 = 2(x - a - b)3$ it is clear that

$$x - 2a + x - 2b = 2x - 2a - 2b$$

$$= 2(x - a - b)$$

Now the expression,

$x^3 - 6x^2a + 12xa^2 - 8a^3 + x^3 - 6x^2b + 12xb^2 - 8b^3$

$= 2(x^3 - 3x^2a - 3x^2b + 3xa^2 + 3xb^2 + 6axb - a^3 - 3a^2b - 3ab^2 - b^3)$

$= 2x^3 - 6x^2a - 6x^2b + 6xa^2 + 6xb^2 + 12xab - 2a^3 - 6a^2b - 6ab^2 - 2b^3$

cancel the common terms on both sides

$12xa^2 + 12xb^2 - 8a^3 - 8b^3$

$= 6xa^2 + 6xb^2 + 12xab - 2a^3 - 6a^2b - 6ab^2 - 2b^3$

$6xa^2 + 6xb^2 - 12xab = 6a^3 + 6b^3 - 6a^2b - 6ab^2$

$6x (a^2 + b^2 - 2ab) = 6 [a^3 + b^3 - ab (a + b)]$

$x (a - b)^2 = [(a + b) (a^2 + b^2 - ab) - (a + b)ab]$

$= (a + b) (a^2 + b^2 - 2ab)$

$= (a + b) (a - b)^2$

$\therefore \quad x = a + b$

Solve the following using "Sunyam Samuccaye" process :

1. $(x - 3)^3 + (x - 9)^3 = 2 (x - 6)^3$
2. $(x + 4)^3 + (x - 10)^3 = 2 (x - 3)^3$
3. $(x + a + b - c)^3 + (x + b + c - a)^3 = 2 (x + b)^3$

EXAMPLE :

$$\frac{(x+2)^3}{(x+3)^3} = \frac{x+1}{x+4}$$

with the text book procedures we proceed as follows

$$\frac{x^3+6x^2+12x+8}{x^3+9x^2+27x+27} = \frac{x+1}{x+4}$$

Now by cross multiplication,

$(x + 4) (x^3 + 6x^2 + 12x + 8) = (x + 1) (x^3 + 9x^2 + 27x + 27)$

$x^4 + 6x^3 + 12x^2 + 8x + 4x^3 + 24x^2 + 48x + 32$

$= x^4 + 9x^3 + 27x^2 + 27x + x^3 + 9x^2 + 27x + 27$

$$x^4 + 10x^3 + 36x^2 + 56x + 32 = x^4 + 10x^3 + 36x^2 + 54x + 27$$

$$56x + 32 = 54x + 27$$

$$56x - 54x = 27 - 32$$

$$2x = -5$$

$$x = -5 / 2$$

Observe that (N1 + D1) with in the cubes on

L.H.S. is $x + 2 + x + 3 = 2x + 5$ and

$N_2 + D_2$ on the right hand side

is $x + 1 + x + 4 = 2x + 5$.

By vedic formula we have $2x + 5 = 0$ $x = - 5 / 2$.

Solve the following by using vedic method :

1. $\dfrac{(x+3)^3}{(x+5)^3} = \dfrac{x+1}{x+7}$
2. $\dfrac{(x-5)^3}{(x-7)^3} = \dfrac{x-3}{x-9}$

Anurupye Sunyamanyat

The Sutra Anurupye Sunyamanyat says : 'If one is in ratio, the other one is zero'.

We use this Sutra in solving a special type of simultaneous simple equations in which the coefficients of 'one' variable are in the same ratio to each other as the independent terms are to each other. In such a context the Sutra says the 'other' variable is zero from which we get two simple equations in the first variable (already considered) and of course give the same value for the variable.

EXAMPLE 1:

$$3x + 7y = 2$$

$$4x + 21y = 6$$

Observe that the y-coefficients are in the ratio 7 : 21 i.e., 1 : 3, which is same as the ratio of independent terms i.e., 2 : 6 i.e., 1 : 3. Hence the other variable $x = 0$ and $7y = 2$ or $21y = 6$ gives $y = 2 / 7$

EXAMPLE 2:

$$323x + 147y = 1615$$

$$969x + 321y = 4845$$

The very appearance of the problem is frightening. But just an observation and anurupye sunyamanyat give the solution $x = 5$, because coefficient of x ratio is

323 : 969 = 1 : 3 and constant terms ratio is

1615 : 4845 = 1 : 3.

$y = 0$ and $323x = 1615$ or $969x = 4845$ gives $x = 5$.

Solve the following by anurupye sunyamanyat.

1. $12x + 78y = 12$
   $16x + 96y = 16$
2. $3x + 7y = 24$
   $12x + 5y = 96$
3. $4x - 6y = 24$
   $7x - 9y = 36$
4. $ax + by = bm$
   $cx + dy = dm$

In solving simultaneous quadratic equations, also we can take the help of the 'sutra' in the following way:

EXAMPLE 3 :

Solve for x and y

$$x + 4y = 10$$

$$x^2 + 5xy + 4y^2 + 4x - 2y = 20$$

$x^2 + 5xy + 4y^2 + 4x - 2y = 20$ can be written as

$(x + y)(x + 4y) + 4x - 2y = 20$

$10(x + y) + 4x - 2y = 20$ (Since $x + 4y = 10$)

$10x + 10y + 4x - 2y = 20$

$14x + 8y = 20$

Now $x + 4y = 10$

$14x + 8y = 20$ and $4 : 8 :: 10 : 20$

from the Sutra, $x = 0$ and $4y = 10$, i.e., $8y = 20$ $y = 10/4 = 2\frac{1}{2}$

Thus $x = 0$ and $y = 2\frac{1}{2}$ is the solution.

Sankalana—Vyavakalanabhyam

This Sutra means 'by addition and by subtraction'. It can be applied in solving a special type of simultaneous equations where the x - coefficients and the y - coefficients are found interchanged.

EXAMPLE 1:

$45x - 23y = 113$

$23x - 45y = 91$

In the conventional method we have to make equal either the coefficient of x or coefficient of y in both the equations. For that we have to multiply equation (1) by 45 and equation (2) by 23 and subtract to get the value of x and then substitute the value of x in one of the equations to get the value of y or we have to multiply equation (1) by 23 and equation (2) by 45 and then subtract to get value of y and then substitute the value of y in one of the equations, to get the value of x. It is difficult process to think of.

From Sankalana - vyavakalanabhyam

add them,

i.e., $(45x - 23y) + (23x - 45y) = 113 + 91$

i.e., $68x - 68y = 204$

$\therefore x - y = 3$

Subtract one from other,

i.e., $(45x - 23y) - (23x - 45y) = 113 - 91$

i.e., $22x + 22y = 22$

$\therefore x + y = 1$

and repeat the same ṣutra, we get $x = 2$ and $y = -1$

Very simple addition and subtraction are enough, however big the coefficients may be.

EXAMPLE 2:

$$1955x - 476y = 2482$$

$$476x - 1955y = -4913$$

Oh ! what a problem ! And still

just add, $2431(x - y) = -2431 \quad x - y = -1$

subtract, $1479(x + y) = 7395 \quad x + y = 5$

once again add, $2x = 4 \quad x = 2$

subtract $-2y = -6 \quad y = 3$

Solve the following problems using Sankalana - Vyavakalanabhyam.

1. $3x + 2y = 18$
   $2x + 3y = 17$
2. $5x - 21y = 26$
   $21x - 5y = 26$
3. $659x + 956y = 4186$
   $956x + 659y = 3889$

Puranapuranabhyam

The Sutra can be taken as Purana — Apuranabhyam which means by the completion or non - completion. Purana is well known in the present system. We can see its application in solving the roots for general form of quadratic equation.

We have : $ax^2 + bx + c = 0$

$$x^2 + (b/a)x + c/a = 0 \text{ (dividing by a)}$$

$$x^2 + (b/a)x = - c/a$$

completing the square (i.e., purana) on the L.H.S.

$$x^2 + (b/a)x + (b^2/4a^2) = -c/a + (b^2/4a^2)$$

$$[x + (b/2a)]^2 = (b^2 - 4ac) / 4a^2$$

Proceeding in this way we finally get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now we apply purana to solve problems.

EXAMPLE 1:

$$x^3 + 6x^2 + 11 x + 6 = 0.$$

Since $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$

Add $(x + 2)$ to both sides

We get

$x^3 + 6x^2 + 11x + 6 + x + 2 = x + 2$

i.e., $x^3 + 6x^2 + 12x + 8 = x + 2$

i.e., $(x + 2)^3 = (x + 2)$

this is of the form $y^3 = y$ for $y = x + 2$

solution $y = 0, y = 1, y = - 1$

i.e., $x + 2 = 0, 1, -1$

which gives $x = -2, -1, -3$

EXAMPLE 2:

$x^3 + 8x^2 + 17x + 10 = 0$

We know $(x + 3)^3 = x^3 + 9x^2 + 27x + 27$

So adding on the both sides, the term $(x^2 + 10x + 17)$, we get

$x^3 + 8x^2 + 17x + x^2 + 10x + 17 = x^2 + 10x + 17$

i.e., $x^3 + 9x^2 + 27x + 27 = x^2 + 6x + 9 + 4x + 8$

i.e., $(x + 3)^3 = (x + 3)^2 \div 4\,(x + 3) - 4$

$y^3 = y^2 + 4y - 4$ for $y = x + 3$

$y = 1, 2, -2.$

Hence $x = -2, -1, -5$

Thus purana is helpful in factorization.

Further purana can be applied in solving Biquadratic equations also.

Solve the following using purana - apuranabhyam.

1. $x^3 - 6x^2 + 11x - 6 = 0$
2. $x^3 + 9x^2 + 23x + 15 = 0$
3. $x^2 + 2x - 3 = 0$
4. $x^4 + 4x^3 + 6x^2 + 4x - 15 = 0$

Calana—Kalanabhyam

In the book on Vedic Mathematics Sri Bharati Krishna Tirthaji mentioned the Sutra 'Calana - Kalanabhyam' at only two places. The Sutra means 'Sequential motion'.

(i) In the first instance it is used to find the roots of a quadratic equation $7x^2 - 11x - 7 = 0$. Swamiji called the sutra as calculus formula. Its application at that point is as follows. Now by calculus formula we say:

$14x - 11 = \pm \sqrt{317}$

A Note follows saying every Quadratic can thus be broken down into two binomial factors. An explanation in terms of first differential, discriminant with sufficient number of examples are given under the chapter 'Quadratic Equations'.

(ii) At the Second instance under the chapter 'Factorization and Differential Calculus' for factorizing expressions of 3rd, 4th and 5th degree, the procedure is mentioned as 'Vedic Sutras relating to Calana - Kalana - Differential Calculus'.

Further other Sutras 10 to 16 mentioned below are also used to get the required results. Hence the sutra and its various applications will be taken up at a later stage for discussion.

But sutra - 14 is discussed immediately after this item.

Now the remaining sutras :

10. YAVADUNAM (The deficiency)
11. VYASTISAMASTIH (Whole as one and one as whole)
12. SESANYAN KENA CARAMENA (Remainder by the last digit)
13. SOPANTYADVAYAMANTYAM (Ultimate and twice the penultimate)
15. GUNITASAMUCCAYAH (The whole product is the same)
16. GUNAKA SAMUCCAYAH (Collectivity of multipliers)

The Sutras have their applications in solving different problems in different contexts. Further they are used along with other Sutras. So it is a bit of inconvenience to deal each Sutra under a separate heading exclusively and also independently. Of course they will be mentioned and also be applied in solving the problems in the forth coming chapter wherever necessary.

This decision has been taken because up to now, we have treated each Sutra independently and have not continued with any other Sutra even if it is necessary. When the need for combining Sutras for filling the gaps in the process arises, we may opt for it. Now we shall deal the fourteenth Sutra, the Sutra left so far untouched.

Ekanyunena Purvena

The Sutra Ekanyunena purvena comes as a Sub-sutra to Nikhilam which gives the meaning 'One less than the previous' or 'One less than the one before'.

(1) The use of this sutra in case of multiplication by 9,99,999.. is as follows .

Method :

(a) The left hand side digit (digits) is (are) obtained by applying the ekanyunena purvena i.e. by deduction 1 from the left side digit (digits) .

e.g. (i) 7 × 9; 7 - 1 = 6 (L.H.S. digit)

(b) The right hand side digit is the complement or difference between the multiplier and the left hand side digit (digits) . i.e. 7 × 9 R.H.S is 9 - 6 = 3.

(c) The two numbers give the answer; i.e. 7 × 9 = 63.

EXAMPLE 1:

8 × 9 Step (a) gives 8 - 1 = 7 (L.H.S. Digit)

Step (b) gives 9 - 7 = 2 (R.H.S. Digit)

Step (c) gives the answer 72

EXAMPLE 2:

15 × 99 Step (a) : 15 - 1 = 14

Step (b) : 99 - 14 = 85 (or 100 - 15)

Step (c) : 15 x 99 = 1485

EXAMPLE 3:

24 × 99

Answer :

$$\begin{array}{c|c} (24-1) & (99-23) \\ =23 & =76(\text{or } 100-24) \end{array} = 2376$$

EXAMPLE 4:

356 × 999

Answer :

$$\begin{array}{c|c} (356-1) & (999-335) \\ =355 & =644 \end{array} = 355644$$

EXAMPLE 5:

878 × 9999

Answer :

$$\begin{array}{c|c} (878-1) & (9999-877) \\ =877 & =9122(10000-876) \end{array} = 8779122$$

Note the Process : The multiplicand has to be reduced by 1 to obtain the LHS and the rightside is mechanically obtained by the subtraction of the L.H.S from the multiplier which is practically a direct application of Nikhilam Sutra.

Now by Nikhilam

$$\begin{array}{r} 24 - 1 = 23 \text{ L.H.S.} \\ \times\ 99 - 23 = 76 \text{ R.H.S. } (100\text{-}24) \\ \hline 23 \ / \ 76 = 2376 \end{array}$$

Reconsider the Example 4:

$$\begin{array}{r} 356 - 1 = 355 \text{ L.H.S.} \\ \times\ 999 - 355 = 644 \text{ R.H.S.} \\ \hline 355 \ / \ 644 = 355644 \end{array}$$

and in Example 5: 878 x 9999 we write

$$\begin{array}{r} 0878 - 1 = 877 \text{ L.H.S.} \\ \times\ 9999 - 877 = 9122 \text{ R.H.S.} \\ \hline 877 \ / \ 9122 = 8779122 \end{array}$$

ALGEBRAIC PROOF :

As any two digit number is of the form (10x + y), we proceed

$(10x + y) \times 99$

$= (10x + y) \times (100 - 1)$

$= 10x \cdot 10^2 - 10x + 10^2 \cdot y - y$

$= x \cdot 10^3 + y \cdot 10^2 - (10x + y)$

$= x \cdot 10^3 + (y - 1) \cdot 10^2 + [\ 10^2 - (10x + y)]$

Thus the answer is a four digit number whose 1000th place is x, 100th place is (y - 1) and the two digit number which makes up the 10th and unit place is the number obtained by subtracting the multiplicand from 100 (or apply nikhilam).

Thus in 37 × 99. The 1000th place is x i.e. 3

100th place is (y - 1) i.e. (7 - 1) = 6

Number in the last two places 100-37=63.

Hence answer is 3663.

Apply Ekanyunena purvena to find out the products

1. 64 × 99
2. 723 × 999
3. 3251 × 9999
4. 43 × 999
5. 256 × 9999
6. 1857 × 99999

We have dealt the cases:

i) When the multiplicand and multiplier both have the same number of digits

ii) When the multiplier has more number of digits than the multiplicand.

In both the cases the same rule applies. But what happens when the multiplier has lesser digits?

i.e. for problems like 42 × 9, 124 × 9, 26325 × 99 etc.,

For this let us have a re-look in to the process for proper understanding.

Multiplication table of 9.

	a	b
2 x 9 =	1	8
3 x 9 =	2	7
4 x 9 =	3	6
8 x 9 =	7	2
9 x 9 =	8	1
10 x 9 =	9	0

Observe the left hand side of the answer is always one less than the multiplicand (here multiplier is 9) as read from Column (a) and the right hand side of the answer is the complement of the left hand side digit from 9 as read from Column (b)

Multiplication table when both multiplicand and multiplier are of 2 digits.

	a	b
11 × 99 =	10	89 = (11-1) / 99 - (11-1) = 1089
12 × 99 =	11	88 = (12-1) / 99 - (12-1) = 1188
13 × 99 =	12	87 = (13-1) / 99 - (13-1) = 1287
18 × 99 =	17	82
19 × 99 =	18	81
20 × 99 =	19	80 = (20-1) / 99 - (20-1) = 1980

The rule mentioned in the case of above table also holds good here.

Further we can state that the rule applies to all cases, where the multiplicand and the multiplier have the same number of digits.

Consider the following Tables.

		a	b
(i)			
	11 × 9 =	9	9
	12 × 9 =	10	8
	13 × 9 =	11	7

---

	18 × 9 =	16	2
	19 × 9 =	17	1
	20 × 9 =	18	0
(ii)	21 × 9 =	18	9
	22 × 9 =	19	8
	23 × 9 =	20	7

---

	28 × 9 =	25	2
	29 × 9 =	26	1
	30 × 9 =	27	0
(iii)	35 × 9 =	31	5
	46 × 9 =	41	4
	53 × 9 =	47	7
	67 × 9 =	60	3

——————— so on.

From the above tables the following points can be observed:

(1) Table (i) has the multiplicands with 1 as first digit except the last one. Here L.H.S of products are uniformly 2 less than the multiplicands. So also with 20 x 9

(2) Table (ii) has the same pattern. Here L.H.S of products are uniformly 3 less than the multiplicands.

(3) Table (iii) is of mixed example and yet the same result i.e. if 3 is first digit of the multiplicand then L.H.S of product is 4 less than the multiplicand; if 4 is first digit of the multiplicand then, L.H.S of the product is 5 less than the multiplicand and so on.

(4) The right hand side of the product in all the tables and cases is obtained by subtracting the R.H.S. part of the multiplicand by Nikhilam.

Keeping these points in view we solve the problems:

EXAMPLE 1 :

42 × 9

(i) Divide the multiplicand (42) of by a Vertical line or by the Sign : into a right hand portion consisting of as many digits as the multiplier.

i.e. 42 has to be written as 4/2 or 4:2

(ii) Subtract from the multiplicand one more than the whole excess portion on the left. i.e. left portion of multiplicand is 4.

one more than it 4 + 1 = 5.

We have to subtract this from multiplicand

i.e. write it as

$$\begin{array}{rcr} 4 & : & 2 \\ & : & -5 \\ \hline 3 & : & 7 \end{array}$$

This gives the L.H.S part of the product.

This step can be interpreted as "take the ekanyunena and sub tract from the previous" i.e. the excess portion on the left.

(iii) Subtract the R.H.S. part of the multiplicand by nikhilam process.

i.e. R.H.S of multiplicand is 2

its nikhilam is 8

It gives the R.H.S of the product

i.e. answer is 3 : 7 : 8 = 378.

Thus 42 × 9 can be represented as

$$\begin{array}{ccccc} 4 & : & 2 & & \\ & : & -5 & : & 8 \\ \hline 3 & : & 7 & : & 8 = 378. \end{array}$$

EXAMPLE 2 :

124 × 9

Here Multiplier has one digit only .

We write 12 : 4

Now step (ii), 12 + 1 = 13

i.e.

$$\begin{array}{ccc} 12 & : & 4 \\ -1 & : & 3 \\ \hline \end{array}$$

Step (iii) R.H.S. of multiplicand is 4. Its Nikhilam is 6

124 × 9 is 12 : 4

$$\begin{array}{ccccc} & & -1 : 3 & : & 6 \\ \hline 11 & : & 1 & : & 6 = 1116 \end{array}$$

The process can also be represented as

124 × 9 = [124 - (12 + 1)] : (10 - 4) = (124 - 13) : 6 = 1116

EXAMPLE 3:

15639 × 99

Since the multiplier has 2 digits, the answer is

[15639 - (156 + 1)] : (100 - 39) = (15639 - 157) : 61
= 1548261

Find the products in the following cases.

1. 58 × 9
2. 62 × 9
3. 427 × 99
4. 832 × 9
5. 24821 × 999
6. 111011 × 99

Ekanyunena Sutra is also useful in Recurring Decimals. We can take up this under a separate treatment.

Thus we have a glimpse of majority of the Sutras. At some places some Sutras are mentioned as Sub-Sutras. Any how we now proceed into the use of Sub-Sutras. As already mentioned the book on Vedic Mathematics enlisted 13 Upa-Sutras.

But some approaches in the Vedic Mathematics book prompted some serious research workers in this field to mention some other Upa-Sutras. We can observe those approaches and developments also.

Anurupyena

The upa-Sutra 'anurupyena' means 'proportionality'. This Sutra is highly useful to find products of two numbers when both of them are near the Common bases i.e powers of base 10 . It is very clear that in such cases the expected 'Simplicity' in doing problems is absent.

EXAMPLE 1:

46 × 43

As per the previous methods, if we select 100 as base we get

$$\begin{array}{ll} 46 & -54 \\ \underline{43} & \underline{-57} \end{array}$$

This is much more difficult and of no use.

Now by 'anurupyena' we consider a working base In three ways. We can solve the problem.

METHOD 1:

Take the nearest higher multiple of 10. In this case it is 50.

Treat it as 100 / 2 = 50. Now the steps are as follows:

(i) Choose the working base near to the numbers under consideration.

i.e., working base is 100 / 2 = 50

(ii) Write the numbers one below the other

i.e.

$$\begin{array}{c} 4\ 6 \\ \underline{4\ 3} \end{array}$$

(iii) Write the differences of the two numbers respectively from 50 against each number on right side

i.e.

$$\begin{array}{ll} 46 & -04 \\ \underline{43} & \underline{-07} \end{array}$$

(iv) Write cross-subtraction or cross- addition as the case may be under the line drawn.

$$\begin{array}{ll} 46 & -04 \\ 43 & -07 \\ \hline (46-7)\text{or} & / \\ (43-4) & / \\ \hline =39 & \end{array}$$

(v) Multiply the differences and write the product in the left side of the answer.

$$\begin{array}{ll} 46 & -04 \\ 43 & -07 \\ \hline \end{array}$$

$$39 \;/\; -4 \times -7$$

$$= 28$$

(vi) Since base is $100 / 2 = 50$, 39 in the answer represents 39×50.

Hence divide 39 by 2 because 50 = 100 / 2

Thus 39 ÷ 2 gives 19½ where 19 is quotient and 1 is remainder.

This 1 as Reminder gives one 50 making the L.H.S of the answer 28 + 50 = 78 (or Remainder ½ × 100 + 28)

i.e. R.H.S 19 and L.H.S 78 together give the answer 1978

We represent it as:

$$\begin{array}{lcc} & 46 & -04 \\ & 43 & -07 \\ \hline 2) & 39 & /\; 28 \\ \hline & 19\tfrac{1}{2} & /\; 28 \end{array}$$

$$= 19 \;/\; 78 = 1978$$

EXAMPLE 2:

42×48.

With 100 / 2 = 50 as working base, the problem is as follows:

$$\begin{array}{lr}
 & 42 \quad -08 \\
 & \underline{48 \quad -02} \\
 & \\
2) & \underline{40 \ / \ 16} \\
 & \\
 & 20 \ / \ 16 \\
\end{array}$$

$$42 \times 48 = 2016$$

METHOD 2:

For the example 1: 46 × 43. We take the same working base 50. We treat it as 50 = 5 × 10. i.e. we operate with 10 but not with 100 as in method

now

$$\begin{array}{ll}
46 & -04 \\
43 & -07 \\ \hline
(46-7)\text{or} & -4\times-7 \\
(43-4) & \quad / \\
=39 & =28 \\ \hline
\end{array}$$

$$\begin{array}{l|l}
39\times 5 & \\
+2 & 8 \\
\text{(carried over} & \\
\end{array}$$

$(195 + 2) / 8 = 1978$

[Since we operate with 10, the R.H.S portion shall have only unit place .Hence out of the product 28, 2 is carried over to left side. The L.H.S portion of the answer shall be multiplied by 5, since we have taken 50 = 5 × 10.]

Now in the example 2: 42 × 48 we can carry as follows by treating 50 = 5 × 10

$$\begin{array}{lcl} 42 & & -08 \\ 48 & & -02 \\ \hline 40 & / & 16 \\ \times 5 & / & \\ \hline 200 & / & 16 \end{array} = 2016$$

METHOD 3:

We take the nearest lower multiple of 10 since the numbers are 46 and 43 as in the first example, We consider 40 as working base and treat it as 4 × 10.

$$\begin{array}{lcl} 46 & & 06 \\ 43 & & 03 \\ \hline (46+3)\text{or} & / & 6 \times 3 \\ (43+6) & / & \\ = 49 & / & = 18 \end{array}$$

Since 10 is in operation 1 is carried out digit in 18.

Since 4 × 10 is working base we consider 49 × 4 on L.H.S of answer i.e. 196 and 1 carried over the left side, giving L.H.S. of answer as 1978. Hence the answer is 1978.

We proceed in the same method for 42 × 48

$$\begin{array}{lcll} 42 & & 02 & \text{working base } 4 \times 10 = 40 \\ 48 & & 08 & \\ \hline 50 & / & 16 & \\ \times 4 & / & & \text{since 10 is in operation} \\ \hline 200 & / & 16 & = 2016 \end{array}$$

Let us see the all the three methods for a problem at a glance.

EXAMPLE 3:

24 × 23

METHOD - 1:

Working base = 100 / 5 = 20

24 04
23 03

5) 27 / 12

5 2/5 / 12 = 5 / 52 = 552

[Since 2 / 5 of 100 is 2 / 5 × 100 = 40 and 40 + 12 = 52]

METHOD - 2:

Working base 2 × 10 = 20

24 04
23 03

27 / 12 it is more convenient than
×2 method – 1 Guss the reason

64 / 12 = 552

Now as 20 itself is nearest lower multiple of 10 for the problem under consideration, the case of method - 3 shall not arise.

Let us take another example and try all the three methods.

EXAMPLE 4:

492 × 404

METHOD - 1 :

Working base = 1000 / 2 = 500

492 -008
404 -096

2)396 / 768 since 1000 is in operation

198 / 768 = 198768

METHOD 2:

Working base = 5 × 100 = 500

$$\begin{array}{l|l} 492 & -008 \\ 404 & -096 \\ \hline 395 & 768 \\ \times 5 & \\ \hline 1980 & 768 \end{array} = 198768$$

METHOD - 3.

Since 400 can also be taken as working base, treat 400 = 4 × 100 as working base.

Thus

$$\begin{array}{l|l} 492 & 092 \\ 404 & 004 \\ \hline 495 & 368 \\ \times 4 & \\ \hline 1984 & 368 \end{array} = 198768$$

since 100 is in operation

No need to repeat that practice in these methods finally takes us to work out all these mentally and getting the answers straight away in a single line.

EXAMPLE 5:

3998 × 4998

Working base = 10000 / 2 = 5000

$$\begin{array}{ll} 3998 & -1002 \\ 4998 & -0002 \\ \hline \end{array}$$

2) 3996 / 2004 since 10,000 is in operation

1998 / 2004 = 19982004

or taking working base = 5 × 1000 = 5,000 and

3998 –1002

4998 –0002 since 100 is carried out

3996 / 2004

×5

19980 / 2004 = 19982004

What happens if we take 4000 i.e. 4 × 1000 as working base?

3998 0002

4998 0998 Since 1000 is an operation

4996 / 1996

As 1000 is in operation, $\overline{1996}$ has to be written as $\overline{1996}$ and 4000 as base, the L.H.S portion 5000 has to be multiplied by 4. i.e. the answer is

4996 / $\overline{1996}$

×4

19982 / 004 =19982004

A simpler example for better understanding.

EXAMPLE 6:

58 × 48

Working base 50 = 5 × 10 gives

50 +8

48 $\overline{2}$

56 / $\overline{16}$

×5

280 / $\overline{16}$ = 280 / 4 = 2784

Since 10 is in operation.

Use anurupyena by selecting appropriate working base and method.

Find the following product.

1. 46 × 46
2. 57 × 57
3. 54 × 45
4. 18 × 18
5. 62 × 48
6. 229 × 230
7. 47 × 96
8. 87965 × 99996
9. 49 × 499
10. 389 × 512

Adyamadyenantya—Mantyena

The Sutra ' adyamadyenantya-mantyena' means 'the first by the first and the last by the last'. Suppose we are asked to find out the area of a rectangular card board whose length and breadth are respectively 6ft . 4 inches and 5 ft. 8 inches. Generally we continue the problem like this.

Area = Length × Breath

= 6' 4" × 5' 8" Since 1' = 12", conversion

= (6 × 12 + 4) (5 × 12 + 8) in to single unit

= 76" 68" = 5168 Sq. inches.

Since 1 sq. ft. =12 × 12 = 144sq.inches we have area

5168 = 144) 5168 (35

144 432

848

720 i.e., 35 Sq. ft 128 Sq. inches

128

By Vedic principles we proceed in the way "the first by first and the last by last"

i.e. 6' 4" can be treated as $6x + 4$ and 5' 8" as $5x + 8$,

Where $x = 1$ ft. = 12 in; x^2 is sq. ft.

Now $(6x + 4)(5x + 8)$

$= 30x^2 + 6.8.x + 4.5.x + 32$

$= 30x^2 + 48x + 20x + 32$

$= 30x^2 + 68.\ x + 32$

$= 30x^2 + (5x + 8).\ x + 32$

Writing $68 = 5 \times 12 + 8$

$= 35x^2 + 8.\ x + 32$

= 35 Sq. ft. + 8 × 12 Sq. in + 32 Sq. in

= 35 Sq. ft. + 96 Sq. in + 32 Sq. in

= 35 Sq. ft. + 128 Sq. in.

It is interesting to know that a mathematically untrained and even uneducated carpenter simply works in this way by mental argumentation. It goes in his mind like this

6' 4"

5' 8"

First by first i.e. 6' × 5' = 30 sq. ft.

Last by last i.e. 4" × 8" = 32 sq. in.

Now cross wise $6 \times 8 + 5 \times 4 = 48 + 20 = 68$.

Adjust as many '12' s as possible towards left as 'units' i.e. $68 = 5 \times 12 + 8$, 5 twelve's as 5 square feet make the first 30+5 = 35 sq. ft ; 8 left becomes 8 × 12 square inches and go towards right i.e. 8 × 12 = 96 sq. in. towards right ives 96+32 = 128sq.in.

Thus he got area in some sort of 35 squints and another sort of 128 sq. units. i.e. 35 sq. ft 128 sq. in

Another Example:

$$\begin{array}{ccc}
 & 4' & 6'' \\
 & 3' & 4'' \\
\hline
4 \times 3 & 4 \times 4 + 6 \times 3 & 6 \times 4 \\
 & = 16 + 18 & \\
= 12 & = 34 & = 24 \\
 & = 2 \times 12 + 10 &
\end{array}$$

Now 12 + 2 = 14, 10 × 12 + 24 = 120 + 24 = 144

Thus 4' 6? × 3' 4? = 14 Sq. ft. 144 Sq. inches.

Since 144 sq. in = 12 × 12 = 1 sq. ft.

The answer is 15 sq. ft.

We can extend the same principle to find volumes of parallelepiped also.

I. Find the area of the rectangles in each of the following situations.

1). l = 3' 8", b = 2' 4 "

2). l = 12' 5" , b = 5' 7"

3). l = 4 yard 3 ft. b = 2 yards 5 ft.(1yard =3ft)

4). l = 6 yard 6 ft. b = 5 yards 2 ft.

II. Find the area of the trapezium in each of the following cases. Recall area = ½ h (a + b) where a, b are parallel sides and h is the distance between them.

1) a = 3' 7", b = 2' 4", h = 1' 5"

2) a = 5' 6", b = 4' 4", h = 3' 2"

3) a = 8' 4", b = 4' 6", h = 5' 1".

Factorization of Quadratics

The usual procedure of factorizing a quadratic is as follows:

$3x^2 + 8x + 4$

$= 3x^2 + 6x + 2x + 4$

$= 3x (x + 2) + 2 (x + 2)$

$= (x + 2) (3x + 2)$

But by mental process, we can get the result immediately. The steps are as follows.

i) Split the middle coefficient in to two such parts that the ratio of the first coefficient to the first part is the same as the ratio of the second part to the last coefficient. Thus we split the coefficient of middle term of $3x^2 + 8x + 4$ i.e. 8 in to two such parts 6 and 2 such that the ratio of the first coefficient to the first part of the middle coefficient i.e. 3:6 and the ratio of the second pat to the last coefficient, i.e. 2: 4 are the same. It is clear that 3:6 = 2:4. Hence such split is valid. Now the ratio 3: 6 = 2: 4 = 1:2 gives one factor x + 2.

ii) Second factor is obtained by dividing the first coefficient of the quadratic by the fist coefficient of the factor already found and the last coefficient of the quadratic by the last coefficient of the factor.

i.e. the second factor is

$$\frac{3x^2}{x} + \frac{4}{2} = 3x + 2$$

Hence $3x^2 + 8x + 4 = (x + 2) (3x + 2)$

E.G.1:

$4x^2 + 12x + 5$

i) Split 12 into 2 and 10 so that as per rule 4 : 2 = 10: 5 = 2 : 1 i.e., 2x + 1 is first factor.

ii) Now

$$\frac{4x^2}{2x} + \frac{5}{1} = 2x + 5 \text{ is second factor.}$$

E.G.2:

$15x^2 - 14xy - 8y^2$

i) Split -14 into -20, 6 so that 15 : - 20 = 3 : - 4 and 6 : - 8 = 3 : - 4. Both are same. i.e., (3x - 4y) is one factor.

ii) Now

$$\frac{15x^2}{3x} + \frac{8y^2}{-4y} = 5x + 2y \text{ is second factor.}$$

Thus $15x^2 - 14xy - 8y^2 = (3x - 4y)(5x + 2y)$.

It is evident that we have applied two sub-sutras 'anurupyena' i.e.'proportionality' and 'adyamadyenantyamantyena' i.e. 'the first by the first and the last by the last' to obtain the above results.

Factorise the following quadratics applying appropriate vedic maths sutras:

1) $3x^2 + 14x + 15$

2) $6x^2 - 23x + 7$

3) $8x^2 - 22x + 5$

4) $12x^2 - 23xy + 10y^2$

Yavadunam Tavadunikrtya Varganca Yojayet

The meaning of the Sutra is 'what ever the deficiency subtract that deficit from the number and write along side the square of that deficit'.

This Sutra can be applicable to obtain squares of numbers close to bases of powers of 10.

METHOD-1 :

Numbers near and less than the bases of powers of 10.

E.G 1:

92 Here base is 10.

The answer is separated in to two parts by a'/'

Note that deficit is 10 - 9 = 1

Multiply the deficit by itself or square it

$1^2 = 1$. As the deficiency is 1, subtract it from the number i.e., 9-1 = 8.

Now put 8 on the left and 1 on the right side of the vertical line or slash i.e., 8/1.

Hence 81 is answer.

E.G. 2:

96^2 Here base is 100.

Since deficit is 100-96=4 and square of it is 16 and the deficiency subtracted from the number 96 gives 96-4 = 92, we get the answer 92 / 16 Thus 962 = 9216.

E.G. 3:

994^2 Base is 1000

Deficit is 1000 - 994 = 6. Square of it is 36.

Deficiency subtracted from 994 gives 994 - 6 = 988

Answer is 988 / 036 [since base is 1000]

E.G. 4:

9988^2 Base is 10,000.

Deficit = 10000 - 9988 = 12.

Square of deficit = 12^2 = 144.

Deficiency subtracted from number = 9988 - 12 = 9976.

Answer is 9976 / 0144 [since base is 10,000].

E.G. 5:

88^2 Base is 100.

Deficit = 100 - 88 = 12.

Square of deficit = 12^2 = 144.

Deficiency subtracted from number = 88 - 12 = 76.

Now answer is 76 / 144 = 7744 [since base is 100]

ALGEBRAIC PROOF:

The numbers near and less than the bases of power of 10 can be treated as (x-y), where x is the base and y, the deficit.

Thus

(1) 9 = (10 -1)

(2) 96 = (100-4)

(3) 994 = (1000-6)

(4) 9988 = (10000-12)

(5) 88 = (100-12)

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$= x\,(x - 2y) + y^2$$

$$= x\,(x - y - y) + y^2$$

$$= \text{Base (number - deficiency)} + (\text{deficit})^2$$

Thus

$$985^2 = (1000 - 15)^2$$

$$= 1000\,(985 - 15) + (15)^2$$

$$= 1000\,(970) + 225$$

$$= 970000 + 225$$

$$= 970225.$$

or we can take the identity $a^2 - b^2 = (a + b)(a - b)$ and proceed as

$$a^2 - b^2 = (a + b)(a - b).$$

gives $a^2 = (a + b)(a - b) + b^2$

Thus for a = 985 and b = 15;

$$a^2 = (a + b)(a - b) + b^2$$

$$985^2 = (985 + 15)(985 - 15) + (15)^2$$

$$= 1000\ (970) + 225$$

$$= 970225.$$

METHOD. 2 :

Numbers near and greater than the bases of powers of 10.

E.G.(1):

13^2.

Instead of subtracting the deficiency from the number we add and proceed as in Method-1.

for 13^2 , base is 10, surplus is 3.

Surplus added to the number = 13 + 3 = 16.

Square of surplus = 32 = 9

Answer is 16 / 9 = 169.

E.G.(2):

112^2

Base = 100, Surplus = 12,

Square of surplus = 122 = 144

add surplus to number = 112 + 12 = 124.

Answer is 124 / 144 = 12544

Or think of identity $a^2 = (a + b)(a - b) + b^2$ for $a = 112$, $b = 12$:

$$112^2 = (112 + 12)(112 - 12) + 122$$

$$= 124\ (100) + 144$$

$$= 12400 + 144$$

$$= 12544.$$

$$
\begin{aligned}
(x + y)^2 &= x^2 + 2xy + y^2 \\
&= x\,(x + 2y) + y^2 \\
&= x\,(x + y + y) + y^2 \\
&= \text{Base (Number + surplus)} + (\text{surplus})^2
\end{aligned}
$$

gives

$$
\begin{aligned}
112^2 &= 100\,(112 + 12) + 122 \\
&= 100\,(124) + 144 \\
&= 12400 + 144 \\
&= 12544.
\end{aligned}
$$

E.G. 3:

10025^2

$= (10025 + 25) / 25^2$

$= 10050 / 0625$ [since base is 10,000]

$= 100500625.$

METHOD 3:

This is applicable to numbers which are near to multiples of 10, 100, 1000 .... etc. For this we combine the upa-Sutra 'anurupyena' and 'yavadunam tavadunikritya varganca yojayet' together.

EXAMPLE 1:

388^2 Nearest base = 400.

We treat 400 as 4 × 100. As the number is less than the base we proceed as follows

Number 388, deficit = 400 - 388 = 12

Since it is less than base, deduct the deficit

i.e. 388 - 12 = 376.

multiply this result by 4 since base is 4 × 100 = 400.

376 × 4 = 1504

Square of deficit = 122 = 144.

Hence answer is 1504 / 144 = 150544 [since we have taken multiples of 100].

EXAMPLE 2:

485^2 Nearest base = 500.

Treat 500 as 5 x 100 and proceed

$$485^2 = \frac{(485-15)}{15^2} \text{ [since deficit is 15]}$$

$$= \frac{470 \times 5}{225} \quad \text{since 500 base is taken as } 5 \times 100$$

$$= \frac{2350}{225} \quad \text{and 2 of 225 is carried over}$$

= 235225.

EXAMPLE 3:

67^2 Nearest base = 70

$$67^2 = \frac{(67-3)}{3^3} \quad \text{dificit is 3}$$

$$= \frac{64 \times 7}{9} \quad [\text{since } 7 \times 10 = 70]$$

$$= \frac{448}{9}$$

= 4489

EXAMPLE 4:

416^2 Nearest (lower) base = 400

Here surplus = 16 and 400 = 4 x 100

$$416^2 = \frac{(416+16)}{16^2} \quad \text{since base is multiple 100,}$$

$= \frac{432\times4}{256}$ and 2 of 256 is carried over

$= \frac{1728}{256}$

= 173056

EXAMPLE 5:

5012^2 Nearest lower base is 5000 = 5 x 1000

Surplus = 12

$5012^2 = \frac{(5012+12)}{12^2}$

$= \frac{5024}{144}$

$= \frac{25120}{144}$

= 25120144.

Apply yavadunam to find the following squares.

1. 72
2. 98^2
3. 987^2
4. 14^2
5. 116^2
6. 1012^2
7. 19^2
8. 475^2
9. 796^2
10. 108^2
11. 9988^2
12. 6014^2.

So far we have observed the application of yavadunam in finding the squares of number. Now with a slight modification yavadunam can also be applied for finding the cubes of numbers.

Cubing of Numbers

EXAMPLE :

Find the cube of the number 106.

We proceed as follows:

i) For 106, Base is 100. The surplus is 6.

Here we add double of the surplus i.e. 106+12 = 118.

(Recall in squaring, we directly add the surplus)

This makes the left-hand -most part of the answer.

i.e. answer proceeds like 118 /.- - - - -

ii) Put down the new surplus i.e. 118-100=18 multiplied by the initial surplus

i.e. 6=108.

Since base is 100, we write 108 in carried over form 108 i.e..

As this is middle portion of the answer, the answer proceeds like 118 / 108 /....

iii) Write down the cube of initial surplus i.e. 63 = 216 as the last portion

i.e. right hand side last portion of the answer.

Since base is 100, write 216 as 216 as 2 is to be carried over.

Answer is 118 / 108 / 216

Now proceeding from right to left and adjusting the carried over, we get the answer

119 / 10 / 16 = 1191016.

E.G.(1):

1023 = (102 + 4) / 6 × 2 / 23

= 106 = 12 = 08

= 1061208.

Observe initial surplus = 2,

next surplus =6 and base = 100.

E.G.(2):

94^3

Observe that the nearest base = 100.

Here it is deficit contrary to the above examples.

i) Deficit = -6. Twice of it -6 × 2 = -12

add it to the number = 94 - 12 = 82.

ii) New deficit is -18.

Product of new deficit x initial deficit = -18 x -6 = 108

iii) deficit 3 = $(-6)^3$ = -216.

Hence the answer is 82 / 108 / $\overline{-216}$

Since 100 is base 1 and -2 are the carried over. Adjusting the carried over in order, we get the answer

(82 + 1) / (08 - 03) / (100 - 16)

= 83 / = 05 / = 84 = 830584

16 becomes 84 after taking 1 from middle most portion i.e. 100. (100-16=84).

Now 08 - 01 = 07 remains in the middle portion, and 2 or 2 carried to it makes the middle as 07 - 02 = 05. Thus we get the above result.

E.G.(3):

998^3 Base = 1000; initial deficit = - 2.

$9983 = (998 - 2 \times 2) / (-6 \times -2) / (-2)^3$

$= 994 / = 012 / = -008$

$= 994 / 011 / 1000 - 008$

$= 994 / 011 / 992$

$= 994011992.$

Find the cubes of the following numbers using yavadunam sutra.

1. 105
2. 114
3. 1003
4. 10007
5. 92
6. 96
7. 993
8. 9991
9. 1000008
10. 999992.

Antyayor Dasake'pi

The Sutra signifies numbers of which the last digits added up give 10. i.e. the Sutra works in multiplication of numbers for example: 25 and 25, 47 and 43, 62 and 68, 116 and 114.

Note that in each case the sum of the last digit of first number to the last digit of second number is 10.

Further the portion of digits or numbers left wards to the last digits remain the same.

At that instant use Ekadhikena on left hand side digits. Multiplication of the last digits gives the right hand part of the answer.

EXAMPLE 1 :

47 × 43

See the end digits sum 7 + 3 = 10 ; then by the sutras antyayor dasakepi and ekadhikena we have the answer.

$$47 \times 43 = (4 + 1) \times 4 \ / \ 7 \times 3$$
$$= 20 \ / \ 21$$
$$= 2021.$$

EXAMPLE 2:

62 x 68

2 + 8 = 10, L.H.S. portion remains the same i.e., 6.

Ekadhikena of 6 gives 7

$$62 \times 68 = (6 \times 7) \ / \ (2 \times 8)$$
$$= 42 \ / \ 16$$
$$= 4216.$$

EXAMPLE 3:

127 × 123

As antyayor dasakepi works, we apply ekadhikena

$$127 \times 123 = 12 \times 13 \ / \ 7 \times 3$$
$$= 156 \ / \ 21$$
$$= 15621.$$

EXAMPLE 4:

65 × 65

We have already worked on this type. As the present sutra is applicable.

We have $65 \times 65 = 6 \times 7 \ / \ 5 \times 5$

$$= 4225.$$

EXAMPLE 5:

395^2

$$\begin{aligned} 395^2 &= 395 \times 395 \\ &= 39 \times 40 \;/\; 5 \times 5 \\ &= 1560 \;/\; 25 \\ &= 156025. \end{aligned}$$

Use Vedic sutras to find the products

1. 125 × 125
2. 34 × 36
3. 98 × 92
4. 401 × 409
5. 693 × 697
6. 1404 × 1406

It is further interesting to note that the same rule works when the sum of the last 2, last 3, last 4 - - - digits added respectively equal to 100, 1000, 10000 -- - - . The simple point to remember is to multiply each product by 10, 100, 1000, - - as the case may be . Your can observe that this is more convenient while working with the product of 3 digit numbers.

E.G. 1:

292 × 208

Here 92 + 08 = 100, L.H.S portion is same i.e. 2

$292 \times 208 = (2 \times 3) \;/\; 92 \times 8$

$\longrightarrow$ 60 / = 736 (for 100 raise the L.H.S. product by 0)

= 60736.

E.G. 2:

848 × 852

Here 48 + 52 = 100, L.H.S portion is 8 and its 'ekhadhikena' is 9.

Now R.H.S product 48 × 52 can be obtained by 'anurupyena' mentally.

$$\begin{array}{lll} & 48 & \bar{2} \\ & 52 & 2 \\ \hline 2) & \underline{50} & \bar{4} \quad = 24 / (100 - 4) \\ & 25 & \quad\quad = 96 \\ & & \quad\quad = 2496 \end{array}$$

and write 848 × 852 = 8 × 9 / 48 × 52

720 = 2496

= 722496.

[Since L.H.S product is to be multiplied by 10 and 2 to be carried over as the base is 100].

E.G. 3:

693 × 607

693 × 607 = 6 × 7 / 93 × 7

= 420 / 651

= 420651.

Find the following products using 'antyayordasakepi'

1. 318 × 312
2. 425 × 475
3. 796 × 744
4. 902 × 998
5. 397 × 393
6. 551 × 549

Antyayoreva

'Atyayoreva' means 'only the last terms'. This is useful in solving simple equations of the following type.

The type of equations are those whose numerator and denominator on the L.H.S. bearing the independent terms stand in the same ratio to each other as the entire numerator and the entire denominator of the R.H.S. stand to each other.

Let us have a look at the following example.

EXAMPLE 1:

$$\frac{x^2+2x+7}{x^2+3x+5}=\frac{x+2}{x+3}$$

In the conventional method we proceed as

$$\frac{x^2+2x+7}{x^2+3x+5}=\frac{x+2}{x+3}$$

$$(x + 3)(x^2 + 2x + 7) = (x + 2)(x^2 + 3x + 5)$$

$$x^3 + 2x^2 + 7x + 3x^2 + 6x + 21$$

$$= x^3 + 3x^2 + 5x + 2x^2 + 6x + 10$$

$$x^3 + 5x^2 + 13x + 21 = x^3 + 5x^2 + 11x + 10$$

Canceling like terms on both sides

$$13x + 21 = 11x + 10$$

$$13x - 11x = 10 - 21$$

$$2x = -11$$

$$x = -11 / 2$$

Now we solve the problem using anatyayoreva.

$$\frac{x^2+2x+7}{x^2+3x+5}=\frac{x+2}{x+3}$$

Consider

$$\frac{x^2+2x+7}{x^2+3x+5}=\frac{x+2}{x+3}$$

Observe that

$$\frac{x^2+2x}{x^2+3x}=\frac{x(x+2)}{x(x+3)}=\frac{x+2}{x+3}$$

This is according to the condition in the sutra. Hence from the sutra

$$\frac{x+2}{x+3}=\frac{7}{5}$$

$$5x + 10 = 7x + 21$$

$$7x - 5x = -21 + 10$$

$$2x = -11$$

$$x = -11 / 2$$

ALGEBRAIC PROOF:

Consider the equation

$$\frac{AC+D}{BC+E}=\frac{A}{B} \qquad \text{...(i)}$$

This satisfies the condition in the sutra since

$$\frac{AC}{BC}=\frac{A}{B}$$

Now cross-multiply the equation (i)

$$B\ (AC + D) = A\ (BC + E)$$

$$BAC + BD = ABC + AE$$

$$BD = AE \text{ which gives}$$

$$\frac{A}{B}=\frac{D}{E} \qquad \text{...(ii)}$$

i.e., the result obtained in solving equation (i) is same as the result obtained in solving equation (ii).

EXAMPLE 2:

Solve

$$\frac{2x^2+3x+10}{3x^2+4x+14}=\frac{2x+3}{3x+4}$$

Since

$$\frac{2x^2+3x}{3x^2+4x}=\frac{x(2x+3)}{x(3x+4)}=\frac{2x+3}{3x+4}$$

We can apply the sutra.

$$\frac{2x+3}{3x+4}=\frac{10}{14}$$

Cross-multiplying

$$28x + 42 = 30x + 40$$

$$28x - 30x = 40 - 42$$

$$-2x = -2 \quad x = -2 / -2 = 1.$$

Let us see the application of the sutra in another type of problem.

EXAMPLE 3:

$$(x + 1)(x + 2)(x + 9) = (x + 3)(x + 4)(x + 5)$$

Re-arranging the equation, we have

$$\frac{(x+1)(x+2)}{(x+4)(x+5)}=\frac{x+3}{x+9}$$

i.e.,

$$=\frac{x^2+3x+2x+3}{x^2+9x+20x+9}$$

Now

$$\frac{x^2+3x}{x^2+9x}=\frac{x(x+3)}{x(x+9)}=\frac{x+3}{x+9}$$

gives the solution by antyayoreva.

Solution is obtained from

$$\frac{x+3}{x+9}=\frac{2}{20}$$

$$20x + 60 = 2x + 18$$

$$20x - 2x = 18 - 60$$

$$18x = -42 \quad x = -42 / 18 = -7 / 3.$$

Once again look into the problem

$(x + 1) (x + 2) (x + 9) = (x + 3) (x + 4) (x + 5)$

Sum of the binomials on each side

$x + 1 + x + 2 + x + 9 = 3x + 12$

$x + 3 + x + 4 + x + 5 = 3x + 12$

It is same. In such a case the equation can be adjusted into the form suitable for application of antyayoreva.

EXAMPLE 4:

$(x + 2) (x + 3) (x + 11) = (x + 4) (x + 5) (x + 7)$

Sum of the binomials on L.H.S. = $3x + 16$

Sum of the binomials on R.H.S. = $3x + 16$

They are same. Hence antyayoreva can be applied. Adjusting we get

$$\frac{(x+2)(x+3)}{(x+4)(x+7)}=\frac{x+5}{x+11}=\frac{2\times 3}{4\times 7}=\frac{6}{28}$$

$$28x + 140 = 6x + 66$$

$$28x - 6x = 66 - 140$$

$$22x = -74$$

$$x = \frac{-74}{22}=\frac{-37}{11}$$

Solve the following problems using 'antyayoreva'

1. $\dfrac{3x^2+5x+8}{5x^2+6x+12}=\dfrac{3x+5}{5x+6}$

2. $\dfrac{4x^2+5x+3}{3x^2+2x+4}=\dfrac{4x+5}{3x+2}$

3. $(x + 3)(x + 4)(x + 6) = (x + 5)(x + 1)(x + 7)$

4. $(x + 1)(x + 6)(x + 9) = (x + 4)(x + 5)(x + 7)$

5. $\dfrac{2x^2+3x+9}{4x^2+5x+17}=\dfrac{2x+3}{4x+5}$

Lopanasthapanabhyam

Lopana sthapanabhyam means 'by alternate elimination and retention'.

Consider the case of factorization of quadratic equation of type $ax^2 + by^2 + cz^2 + dxy + eyz + fzx$ This is a homogeneous equation of second degree in three variables x, y, z. The sub-sutra removes the difficulty and makes the factorization simple. The steps are as follows:

i) Eliminate z by putting z = 0 and retain x and y and factorize thus obtained a quadratic in x and y by means of 'adyamadyena' sutra.;

ii) Similarly eliminate y and retain x and z and factorize the quadratic in x and z.

iii) With these two sets of factors, fill in the gaps caused by the elimination process of z and y respectively. This gives actual factors of the expression.

EXAMPLE 1:

$3x^2 + 7xy + 2y^2 + 11xz + 7yz + 6z^2$.

Step (i): Eliminate z and retain x, y; factorize

$3x^2 + 7xy + 2y^2 = (3x + y)(x + 2y)$

Step (ii): Eliminate y and retain x, z; factorize

$3x^2 + 11xz + 6z^2 = (3x + 2z)(x + 3z)$

Step (iii): Fill the gaps, the given expression

$= (3x + y + 2z)(x + 2y + 3z)$

EXAMPLE 2:

$12x^2 + 11xy + 2y^2 - 13xz - 7yz + 3z^2$.

Step (i): Eliminate z i.e., z = 0; factorize

$12x^2 + 11xy + 2y^2 = (3x + 2y)(4x + y)$

Step (ii): Eliminate y i.e., y = 0; factorize

$12x^2 - 13xz + 3z^2 = (4x - 3z)(3x - z)$

Step (iii): Fill in the gaps; the given expression

$= (4x + y - 3z)(3x + 2y - z)$

EXAMPLE 3:

$3x^2 + 6y^2 + 2z^2 + 11xy + 7yz + 6xz + 19x + 22y + 13z + 20$

Step (i): Eliminate y and z, retain x and independent term i.e., y = 0, z = 0 in the expression (E).

Then $E = 3x^2 + 19x + 20 = (x + 5)(3x + 4)$

Step (ii): Eliminate z and x, retain y and independent term i.e., z = 0, x = 0 in the expression.

Then $E = 6y^2 + 22y + 20 = (2y + 4)(3y + 5)$

Step (iii): Eliminate x and y, retain z and independent term i.e., x = 0, y = 0 in the expression.

Then $E = 2z^2 + 13z + 20 = (z + 4)(2z + 5)$

Step (iv): The expression has the factors (think of independent terms) $= (3x + 2y + z + 4)(x + 3y + 2z + 5)$.

In this way either homogeneous equations of second degree or general equations of second degree in three variables can be very easily solved by applying 'adyamadyena' and 'lopanasthapanabhyam' sutras.

Solve the following expressions into factors by using appropriate sutras:

1. $x^2 + 2y^2 + 3xy + 2xz + 3yz + z^2$.
2. $3x^2 + y^2 - 4xy - yz - 2z^2 - zx$.
3. $2p^2 + 2q^2 + 5pq + 2p - 5q - 12$.
4. $u^2 + v^2 - 4u + 6v - 12$.
5. $x^2 - 2y^2 + 3xy + 4x - y + 2$.
6. $3x^2 + 4y^2 + 7xy - 2xz - 3yz - z^2 + 17x + 21y - z + 20$.

Highest Common Factor

To find the Highest Common Factor i.e. H.C.F. of algebraic expressions, the factorization method and process of continuous division are in practice in the conventional system. We now apply' Lopana - Sthapana' Sutra, the 'Sankalana vyavakalanakam' process and the 'Adyamadya' rule to find out the H.C.F in a more easy and elegant way.

EXAMPLE 1:

Find the H.C.F. of x2 + 5x + 4 and x2 + 7x + 6.

1. Factorization method:

$x^2 + 5x + 4 = (x + 4)(x + 1)$

$x^2 + 7x + 6 = (x + 6)(x + 1)$

H.C.F. is $(x + 1)$.

2. Continuous division process.

$$\begin{array}{llll}
x^2 + 5x + 4) & x^2 + 7x + 6\ (1 & & \\
 & \underline{x^2 + 5x + 4} & & \\
 & 2x + 2) & x^2 + 5x + 4\ (\tfrac{1}{2}x & \\
 & & \underline{x^2 + x} & \\
 & & 4x + 4) & 2x + 2\ (\tfrac{1}{2} \\
 & & & \underline{2x + 2} \\
 & & & 0
\end{array}$$

Thus $4x + 4$ i.e., $(x + 1)$ is H.C.F.

3. Lopana - Sthapana process i.e. elimination and retention or alternate destruction of the highest and the lowest powers is as below:

$$\text{Subtract}\left\{\begin{array}{r} x^2+5x+4 \\ x^2+7x+6 \\ \hline -2)-2x-2 \\ \overline{x+1} \end{array}\right.$$

i.e., (x + 1) is H.C.F

EXAMPLE 2:

Find H.C.F. of $2x^2 - x - 3$ and $2x^2 + x - 6$

$$\text{Subtract}\left\{\begin{array}{l} 2x^2-x-3 \\ 2x^2+x-6 \\ \hline -1)-2x+3 \\ \quad\;\; \overline{2x+3} \text{ is the HCF.} \end{array}\right.$$

EXAMPLE 3:

$x^3 - 7x - 6$ and $x^3 + 8x^2 + 17x + 10$.

Now by Lopana - Sthapana and Sankalana - Vyavakalanabhyam

$$\text{Subtract}\left\{\begin{array}{r} x^3 \qquad\;\; -7x-6 \\ x^3+8x^2+17x+10 \\ \hline -8)-8x^2-24x-16 \\ \overline{x^2+3x+2} \end{array}\right.$$

EXAMPLE 4:

$x^3 + 6x^2 + 5x - 12$ and $x^3 + 8x^2 + 19x + 12$.

$$\begin{array}{l}\text{Add} \\ \div 2x\end{array}\left\{\begin{array}{r} x^3+6x^2+5x-12 \\ x^3+8x^2+19x+12 \\ \hline 2x^3+14x^2+24x \\ \hline x^2+7x+12 \end{array}\right.$$

(or)

$$\begin{array}{r} \text{Subtract} \\ \div -2 \end{array} \left\{ \begin{array}{r} x^3+6x^2+5x-12 \\ \underline{x^3+8x^2+19x+12} \\ \underline{2x^2-14x-24} \\ x^2+7x+12 \end{array} \right.$$

EXAMPLE 5:

$2x^3 + x^2 - 9$ and $x^4 + 2x^2 + 9$.

By Vedic sutras:

Add:$(2x^3 + x^2 - 9) + (x^4 + 2x^2 + 9)$

$= x^4 + 2x^3 + 3x^2.$

$\div\ x^2$ gives

$x^2 + 2x + 3$ ...(i)

Subtract after multiplying the first by x and the second by 2.

Thus$(2x^4 + x^3 - 9x) - (2x^4 + 4x^2 + 18)$

$= x^3 - 4x^2 - 9x - 18$ ...(ii)

Multiply (i) by x and subtract from (ii)

$x^3 - 4x^2 - 9x - 18 - (x^3 + 2x^2 + 3x)$

$= - 6x^2 - 12x - 18$

$\div - 6$ gives $x^2 + 2x + 3$.

Thus $(x^2 + 2x + 3)$ is the H.C.F. of the given expressions.

ALGEBRAIC PROOF:

Let P and Q be two expressions and H is their H.C.F. Let A and B the Quotients after their division by H.C.F.

i.e., $\frac{P}{H} = A$ and $\frac{Q}{H} = B$ which gives $P = A.H$ and $Q = B.H$

$P + Q = AH + BH$ and $P - Q = AH - BH$

$= (A+B).H \qquad\qquad = (A-B).H$

Thus we can write P ± Q = (A ± B).H

Similarly MP = M.AH and NQ = N.BH gives MP ± NQ = H (MA ± NB)

This states that the H.C.F. of P and Q is also the H.C.F. of P±Q or MA±NB.

i.e. we have to select M and N in such a way that highest powers and lowest powers (or independent terms) are removed and H.C.F appears as we have seen in the examples.

Find the H.C.F. in each of the following cases using Vedic sutras:

1. $x^2 + 2x - 8, x^2 - 6x + 8$
2. $x^3 - 3x^2 - 4x + 12, x^3 - 7x^2 + 16x - 12$
3. $x^3 + 6x^2 + 11x + 6, x^3 - x^2 - 10x - 8$
4. $6x^4 - 11x^3 + 16x^2 - 22x + 8,$
   $6x^4 - 11x^3 - 8x^2 + 22x - 8.$

Vilokanam

The Sutra 'Vilokanam' means 'Observation'. Generally we come across problems which can be solved by mere observation. But we follow the same conventional procedure and obtain the solution. But the hint behind the Sutra enables us to observe the problem completely and find the pattern and finally solve the problem by just observation.

Let us take the equation x + (1/x) = 5/2 Without noticing the logic in the problem, the conventional process tends us to solve the problem in the following way.

$$x + \frac{1}{x} = \frac{5}{2}$$

$$\frac{x^2 + 1}{x} = \frac{5}{2}$$

$$2x^2 + 2 = 5x$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x - 2) - (x - 2) = 0$$

$$(x - 2)(2x - 1) = 0$$

$$x - 2 = 0 \text{ gives } x = 2$$

$$2x - 1 = 0 \text{ gives } x = \tfrac{1}{2}$$

But by Vilokanam i.e., observation

$x+\dfrac{1}{x}=\dfrac{5}{2}$ can be viewed as

$x+\dfrac{1}{x}=2+\dfrac{1}{2}$ giving x = 2 or ½.

Consider Some Examples.

EXAMPLE 1 :

$$\frac{x}{x+2}+\frac{x+2}{x}=\frac{34}{15}$$

In the conventional process, we have to take L.C.M, cross-multiplication. simplification and factorization. But Vilokanam gives

$$\frac{34}{15}=\frac{9+25}{5\times3}=\frac{3}{5}+\frac{5}{3}$$

$$\frac{x}{x+2}+\frac{x+2}{x}=\frac{3}{5}+\frac{5}{3}$$

gives

$$\frac{x}{x+2}=\frac{3}{5} \text{ or } \frac{5}{3}$$

$$5x = 3x + 6$$

or $3x = 5x + 10$

$2x = 6$

or $-2x = 10$

$x = 3$

or $x = -5$

EXAMPLE 2 :

$$\frac{x+5}{x+6}+\frac{x+6}{x+5}=\frac{113}{56}$$

Now,

$$\frac{113}{56}=\frac{49+64}{7\times 8}=\frac{7}{8}+\frac{8}{7}$$

$$\frac{x+5}{x+6}=\frac{7}{8} \text{ or } \frac{x+5}{x+6}=\frac{8}{7}$$

$8x + 40 = 7x + 42$ $\quad$ $7x + 35 = 8x + 48$

or

$x = 42 - 40 = 2$ $\quad$ $-x = 48 - 35 = 13$

$x = 2$ $\quad$ or $x = -13.$

EXAMPLE 3:

$$\frac{5x+9}{5x-9}+\frac{5x-9}{5x+9}=2\frac{82}{319}$$

At first sight it seems to a difficult problem.

But careful observation gives

$$2\frac{82}{319}=\frac{720}{319}=\frac{841-121}{11\times 29}=\frac{29}{11}-\frac{11}{29}$$

(Note: $29^2 = 841$, $11^2 = 121$)

$$\frac{5x+9}{5x-9}=\frac{29}{11} \text{ or } \frac{-11}{29}$$

(Note: 29 = 20 + 9 = 5 x 4 + 9 ; 11 = 20 - 9=5 x 4 - 9)

i.e.,

x = 4 or

$$\frac{5x+9}{5x-9}=\frac{-11}{29}$$

$$145x + 261 = -55x + 99$$

$$145x + 55x = 99 - 261$$

$$200x = -162$$

$$x=\frac{-162}{200}=\frac{-81}{100}$$

Simultaneous Quadratic Equations

EXAMPLE 1

x + y = 9 and xy = 14.

We follow in the conventional way that

$(x - y)^2 = (x + y)^2 - 4xy = 9^2 - 4 (14) = 81 - 56 = 25$

$x - y = \sqrt{25} = \pm 5$

$$\left.\begin{array}{l} x+y=9 \\ x-y=5 \end{array}\right\} \text{gives } 2x=14,\ x=7$$

x + y = 9 gives 7 + y = 9

y = 9 - 7 = 2.

Thus the solution is x = 7, y = 2 or x = 2, y = 7.

But by Vilokanam, xy = 14 gives x = 2, y = 7 or x = 7, y = 2 and these two sets satisfy x + y = 9 since 2 + 7 = 9 or 7 + 2 = 9. Hence the solution.

EXAMPLE 2:

$5x - y = 7$ and $xy = 6$.

$xy = 6$ gives $x = 6, y = 1; x = 1, y = 6;$

$x = 2, y = 3; x = 3, y = 2$ and of course negatives of all these.

Observe that $x = 6, y = 1; x = 1, y = 6$: are not solutions because they do not satisfy the equation $5x - y = 7$.

But for $x = 2, y = 3; 5x - y = 5 (2) - 3 = 10 - 3 = 7$ we have $5(3) - 2 \neq 7$.

Hence $x = 2, y = 3$ is a solution.

For $x = 3, y = 2$ we get $5 (3) - 2 = 15 - 2$? 7.

Hence it is not a solution.

Negative values of the above are also not the solutions. Thus one set of the solutions i.e., $x = 2, y = 3$ can be found. Of course the other will be obtained from solving $5x - y = 7$ and $5x + y = -13$.

i.e., $x = -3 / 5, y = -10$.

Partial Fractions

EXAMPLE 1

Resolve

$\frac{2x+7}{(x+3)(x+4)}$ into partial fractions.

We write $\frac{2x+7}{(x+3)(x+4)} = \frac{A}{(x+3)} + \frac{B}{(x+4)}$

$$= \frac{A(x+4)+B(x+3)}{(x+3)(x+4)}$$

$$2x + 7 = A (x + 4) + B (x + 3).$$

We proceed by comparing coefficients on either side

coefficient of x : $A + B = 2$ ....(i) × 3

Independent of x : $4A + 3B = 7$ ....(ii)

Solving (ii) - (i) × 3

$$\begin{array}{r} 4A + 3B = 7 \\ 3A + 3B = 6 \\ \hline A = 1 \end{array}$$

$A = 1$ in (i) gives, $1 + B = 2$ i.e., $B = 1$

Or we proceed as

$$2x + 7 \equiv A(x + 4) + B(x + 3).$$

Put $x = -3$, $\quad 2(-3) + 7 \equiv A(-3 + 4) + B(-3 + 3)$

$$1 = A(1) \therefore A = 1.$$

$x = -4$, $\quad 2(-4) + 7 = A(-4 + 4) + B(-4 + 3)$

$$-1 = B(-1) \therefore B = 1.$$

Thus $\dfrac{2x+7}{(x+3)(x+4)} = \dfrac{1}{(x+3)} + \dfrac{1}{(x+4)}$

But by Vilokanam $\dfrac{2x+7}{(x+3)(x+4)}$ can be resolved as

$(x + 3) + (x + 4) = 2x + 7$, directly we write the answer.

EXAMPLE 2:

$$\frac{3x+13}{(x+1)(x+2)}$$

from $(x + 1)$, $(x + 2)$ we can observe that

$10(x + 2) - 7(x + 1) = 10x + 20 - 7x - 7 = 3x + 13$

Thus $\dfrac{3x+13}{(x+1)(x+2)} = \dfrac{10}{x+1} - \dfrac{7}{x+2}$

EXAMPLE 3:

$$\frac{9}{x^2+x-2}$$

As $x^2 + x - 2 = (x - 1)(x + 2)$ and

$$9 = 3(x + 2) - 3(x - 1)$$

$$(3x + 6 - 3x + 3 = 9)$$

We get by Vilokanam, $\frac{9}{x^2+x-2}=\frac{3}{x-1}-\frac{3}{x+2}$

I. Solve the following by mere observation i.e. vilokanam

1. $x+\frac{1}{x}=\frac{25}{12}$
2. $x-\frac{1}{x}=\frac{5}{6}$
3. $\frac{x}{x+1}+\frac{x+1}{x}=9\frac{1}{9}$
4. $\frac{x+7}{x+9}-\frac{x+9}{x+7}=\frac{32}{63}$

II. Solve the following simultaneous equations by vilokanam.

1. $x - y = 1,\ xy = 6$
2. $x + y = 7,\ xy = 10$
3. $2x + 3y = 19,\ xy = 15$
4. $x + y = 4,\ x^2 + xy + 4x = 24.$

III. Resolve the following into partial fractions.

1. $\frac{2x-5}{(x-2)(x-3)}$

2. $\frac{9}{(x+1)(x-2)}$

3. $\frac{x-13}{x^2-2x-15}$

4. $\frac{3x+4}{3x^2+3x+2}$

Gunìta Samuccayah—Samuccaya Gunìtah

In connection with factorization of quadratic expressions a sub-Sutra, viz. 'Gunita samuccayah-Samuccaya Gunitah' is useful. It is intended for the purpose of verifying the correctness of obtained answers in multiplications, divisions and factorizations. It means in this context:

'The product of the sum of the coefficients sc in the factors is equal to the sum of the coefficients sc in the product'.

Symbolically we represent as sc of the product = product of the sc (in the factors).

EXAMPLE 1:

$(x + 3)(x + 2) = x^2 + 5x + 6$

Now $(x + 3)(x + 2) = 4 \times 3 = 12$: Thus verified.

EXAMPLE 2:

$(x - 4)(2x + 5) = 2x^2 - 3x - 20$

Sc of the product $2 - 3 - 20 = -21$

Product of the Sc $= (1 - 4)(2 + 5) = (-3)(7) = -21$.

Hence verified.

In case of cubics, biquadratics also the same rule applies.

We have $(x + 2)(x + 3)(x + 4) = x^3 + 9x^2 + 26x + 24$

Sc of the product $= 1 + 9 + 26 + 24 = 60$

Product of the Sc $= (1 + 2)\,(1 + 3)\,(1 + 4)$

$= 3 \times 4 \times 5 = 60$. Verified.

EXAMPLE 3:

$(x + 5)\,(x + 7)\,(x - 2) = x^3 + 10x^2 + 11x - 70$

$(1 + 5)\,(1 + 7)\,(1 - 2) = 1 + 10 + 11 - 70$

i.e., 6×8 x $-1 = 22 - 70$

i.e., $-48 = -48$ Verified.

We apply and interpret So and Sc as sum of the coefficients of the odd powers and sum of the coefficients of the even powers and derive that So = Sc gives $(x + 1)$ is a factor for thee concerned expression in the variable x. Sc = 0 gives $(x - 1)$ is a factor.

Verify whether the following factorization of the expressions are correct or not by the Vedic check:

i.e. Gunita. Samuccayah-Samuccaya Gunitah:

1. $(2x + 3)\,(x - 2) = 2x^2 - x - 6$
2. $12x^2 - 23xy + 10y^2 = (3x - 2y)\,(4x - 5y)$
3. $12x^2 + 13x - 4 = (3x - 4)\,(4x + 1)$
4. $(x + 1)\,(x + 2)\,(x + 3) = x^3 + 6x^2 + 11x + 6$
5. $(x + 2)\,(x + 3)\,(x + 8) = x^3 + 13x^2 + 44x + 48$

So far we have considered a majority of the upa-sutras as mentioned in the Vedic mathematics book. Only a few Upa-Sutras are not dealt under a separate heading . They are:

(2) S'ISYATE S'ESASAMJ ÑAH

(4) KEVALAIH SAPTAKAMGUNYAT

(5) VESTANAM

(6) YAVADUNAM TAVADUNAM and

(10) SAMUCCAYAGUNITAH already find place in respective places.

Further in some other books developed on Vedic Mathematics DVANDAYOGA, SUDHA, DHVAJANKAM are also given as Sub-Sutras. They are mentioned in the Vedic Mathematics text also. But the list in the text (by the Editor) does not contain them. We shall also discuss them at appropriate places, with these three included, the total number of upa-Sutras comes to sixteen.

We now proceed to deal the Sutras with reference to their variety, applicability, speed, generality etc. Further we think how 'the element of choice in the Vedic system, even of innovation, together with mental approach, brings a new dimension to the study and practice of Mathematics. The variety and simplicity of the methods brings fun and amusement, the mental practice leads to a more agile, alert and intelligent mind and innovation naturally follow' (Prof. K.R.Williams, London).

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ISBN 978-81-261-3504-2

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